

# PhD Open Days



## Non-Abelian Geometric Phases for Polarization Control in Waveguides

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### Motivation

Manipulating the polarization of light is fundamental to a wide range of photonic and quantum technologies<sup>1,2</sup>. Geometric phases arising in deformed circular guides provide a mechanism for polarization control of the propagating waves<sup>3</sup>. However, such control is limited by the Abelian character of the Berry connection governing the geometric phases.

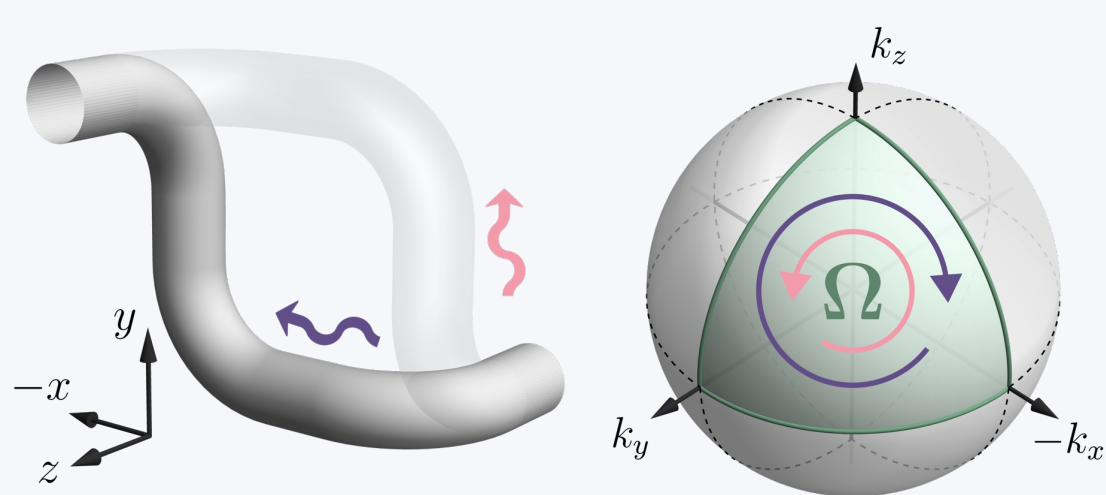
### Abstract

We show that a matrix-valued Berry connection governs the polarization of propagating waves in adiabatically deformed waveguides with a doubly degenerate fundamental mode. The effect of deformations can be visualized as a sequence of rotations of the polarization state on the Bloch sphere. Within this framework, we compare the polarization control enabled by Abelian and non-Abelian connections. **Key Findings:**

| Connection                          | Abelian  | Non-Abelian |
|-------------------------------------|----------|-------------|
| Polarization of resonator modes     | Constant | Variable    |
| Deformation-invariant polarizations | Yes      | No          |

### 1. Circular Guides

- Adiabatic deformations producing closed paths in normalized wave vector space.
- Geometric phases attained by eigenmodes, RCP(+) and LCP(-), are insensitive to deformations order; only depend on solid angle  $\Omega$ :  $\mathbf{E}_{\pm}^{\text{out}} = \mathbf{E}_{\pm}^{\text{in}} \exp(\mp i\Omega)$ .

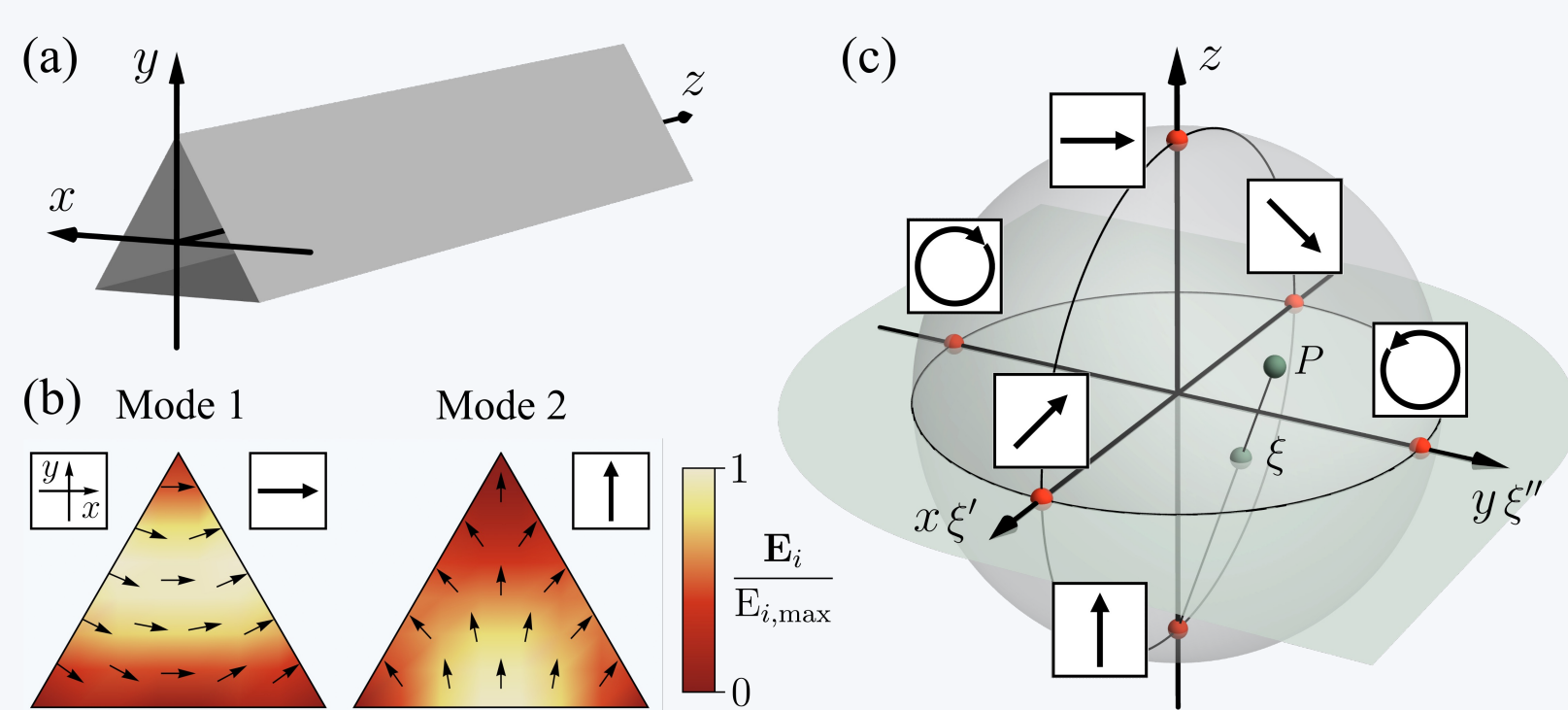


Can the geometric phase depend on the order of deformations?

**New degrees of freedom for polarization control**

### 2. Triangular Guides

- Hollow metallic waveguide with a cross section in the shape of an equilateral triangle<sup>(a)</sup>.
- Two degenerate fundamental modes w/ orthogonal average polarizations [black arrows in the inset frames]<sup>(b)</sup>.



- Polarization of mode superposition  $\mathbf{p}$  is determined by a single complex number  $\xi$ :

$$\mathbf{E} = c_1 \mathbf{E}_1 + c_2 \mathbf{E}_2 : \mathbf{p} \sim \hat{\mathbf{x}} + \xi \hat{\mathbf{y}}, \xi = c_2/c_1$$

- Stereographic projection of  $\xi$  maps polarization states onto the Bloch sphere<sup>(c)</sup>.

### 3. Deformation Action on the Bloch Sphere

- Adiabatically-deformed guide.
- Superposition of degenerate local fundamental modes  $\psi_1$  and  $\psi_2$ .

$$\psi(s) = c_1(s)\psi_1(s) + c_2(s)\psi_2(s)$$

$s = \text{arc length}$

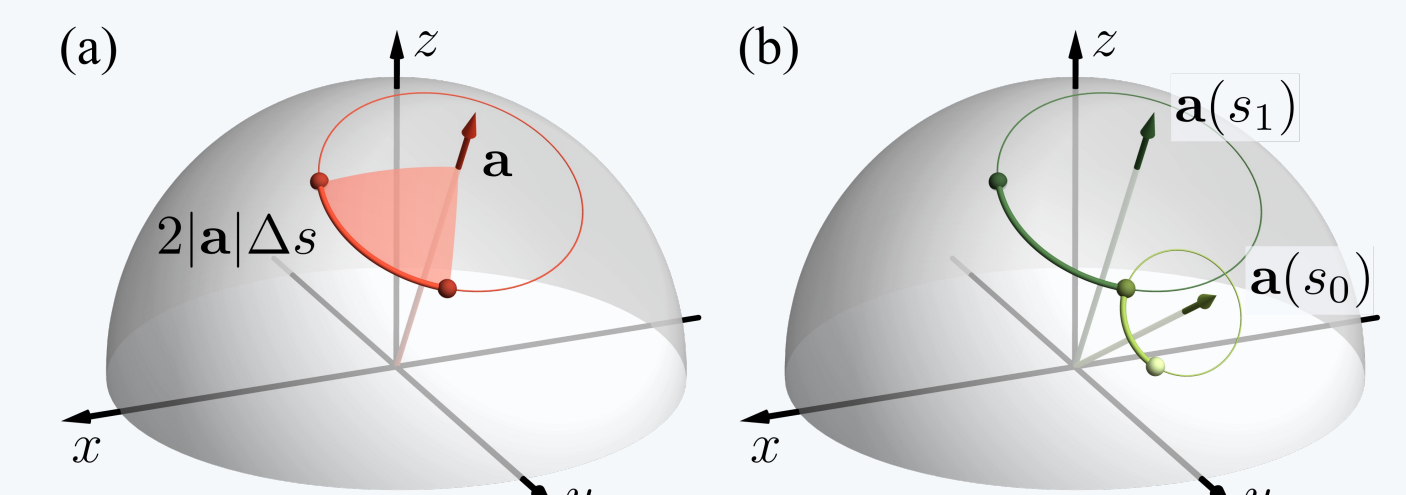
- Coefficients after propagation length  $l$ :

$$(c_1(l), c_2(l))^T = T \cdot (c_1(0), c_2(0))^T$$

- Transmission matrix  $T$  is an ordered product of exponentials of the  $2 \times 2$  matrix-valued Berry connection  $\mathbf{A}$  with elements  $A_{ij} = i \langle \psi_i | \partial_s \psi_j \rangle$ .

$$T = \lim_{N \rightarrow \infty} \prod_{n=0}^N \exp(i\mathbf{A}(s_n)\Delta s), \Delta s = l/N$$

- Matrix exponential of  $\mathbf{A}$  represents a clockwise rotation on the Bloch sphere, about the axis  $\mathbf{a} = \text{tr}(\mathbf{A}\boldsymbol{\sigma})/2$  and by an angle  $2|\mathbf{a}|\Delta s$ <sup>(a)</sup>.



- Matrix  $T$  is a series of infinitesimal rotations on the Bloch sphere<sup>(b)</sup>

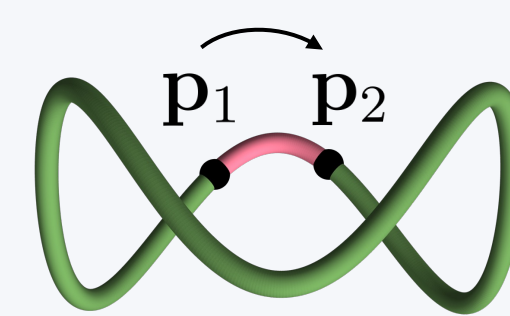
$$T \sim \exp(i\mathbf{A}(s_1)\Delta s) \times \exp(i\mathbf{A}(s_0)\Delta s)$$

- Rotations commute iff connection is Abelian:

Sensitivity to order of deformations requires a **non-Abelian** connection

### 4. Resonator Modes

- Closed guides: deformation path is a loop pinned at merged ports. Pin is redefinable:



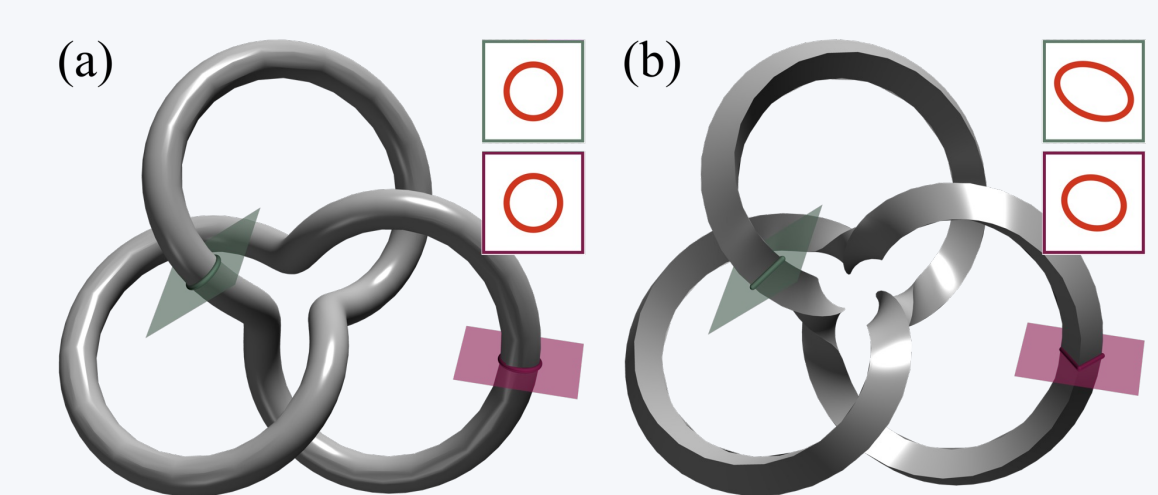
$$\mathbf{U}(\mathbf{p}_i) = \exp(i\mathbf{A}(\mathbf{p}_i)\Delta s)$$

$$T(\mathbf{p}_1) \sim \mathbf{U}(\mathbf{p}_1) \times \dots \times \mathbf{U}(\mathbf{p}_2)$$
$$T(\mathbf{p}_2) \sim \mathbf{U}(\mathbf{p}_2) \times \dots \times \mathbf{U}(\mathbf{p}_1)$$

- $T$ 's differ iff  $\mathbf{A}$  is non-Abelian and eigenvectors give polarization profile of resonator modes.

Polarization is **(in)constant** if  $\mathbf{A}$  is **(non-)Abelian**

- $\mathcal{O}^{(a)}(\Delta^{(b)})$  section has Abelian (non-Abelian)  $\mathbf{A}$ .

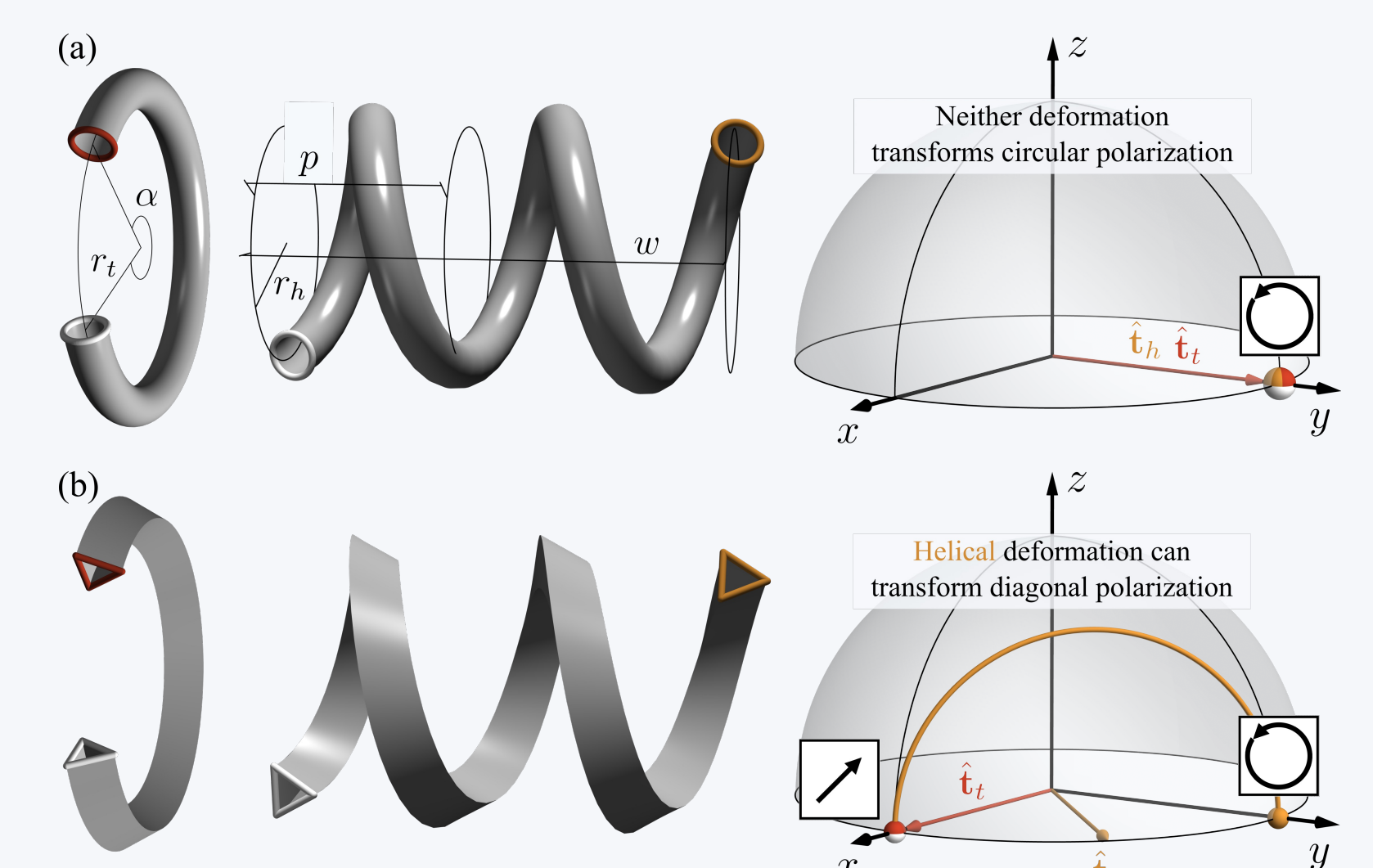


### 5. Conserved Polarizations

- $T$  can be seen as one effective rotation:  $T = \exp(it\hat{\mathbf{t}} \cdot \boldsymbol{\sigma})$ . Intersections of  $\hat{\mathbf{t}}$  with the Bloch sphere are **constant polarizations**.

Deformation-dependent iff  $\mathbf{A}$  is non-Abelian

- Diagonal-to-circular polarization conversion via **toroidal** and **helical** deformations:
- Impossible** w/ circular guides: both axes intersect the RCP state<sup>(a)</sup>.
- Possible** w/ triangular guides: **helical** axis intersects no relevant polarization<sup>(b)</sup>.



### References

<sup>1</sup>H. Zafar and M. F. Pereira, *Laser Photonics Rev* **18**, 2301025 (2024).

<sup>2</sup>F. Flamini et al., *Rep Prog Phys* **82**, 016001 (2019).

<sup>3</sup>S. A. H. Gangaraj et al., *IEEE J Multiscale Multiphys Comput Tech* **2**, 3 (2017).

<sup>4</sup>F. Wilczek and A. Zee, *Phys Rev B* **52**, 2111 (1984).

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