PhD Open Days

Quantum Computing Strong-Field QED

APPLAuSE PhD programme
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Understanding wave propagation and instabilities in plasmas is central to both the fundamental and applied sides of plasma physics, including open questions in astro-plasmas and controlled nuclear fusion engineering. A key quantity of study is the dispersion relation, describing how perturbations of frequency and wavenumber $\omega(k)$ propagate or grow. Conventional extraction of dispersion branches relies on root-finding, which can become numerically unstable or costly, especially when many intersecting modes are present. Quantum Computing could provide an alternative route to obtain information about these instabilities.

Motivation

- Quantum algorithms have the potential to solve certain classes of eigenvalue problems more efficiently than their classical counterparts.
- While methods such as Quantum Phase Estimation and the Rodeo Algorithm [1] efficiently recover real spectra of Hermitian Hamiltonians, most instabilities require access to complex eigenvalues.
- To this end, we explore a variational framework [2] that encodes non-Hermitian plasma Hamiltonians into measurable Hermitian cost functions, whose minima yield the desired eigenvalues.
- As a proof of concept, we apply both classes of algorithms to simple plasma dispersion problems with known analytical solutions.

Hermitian Hamiltonian - Landau Damping

One of the most well-known plasma instabilities is Landau Damping. It has been shown [3,4] that the 0D1V Electrostatic Vlasov+Ampère, Fourier version of this problem can be written in Hermitian Hamiltonian form.

This allows 2 methods to extract the dispersion relation: through the Rodeo Algorithm [1], or post-processing the expectation value of the electric field after standard Exact Diagonalization Hamiltonian simulation.

$$\hat{H} = \Sigma_{j=0}^{N_v-1}\, v_j\, k\,|j\rangle\langle j| + v_j\, \alpha_j(|j\rangle\langle N_v| + |N_v\rangle\langle j|) = \hat{H}_v + \hat{H}_l + \hat{H}_c$$

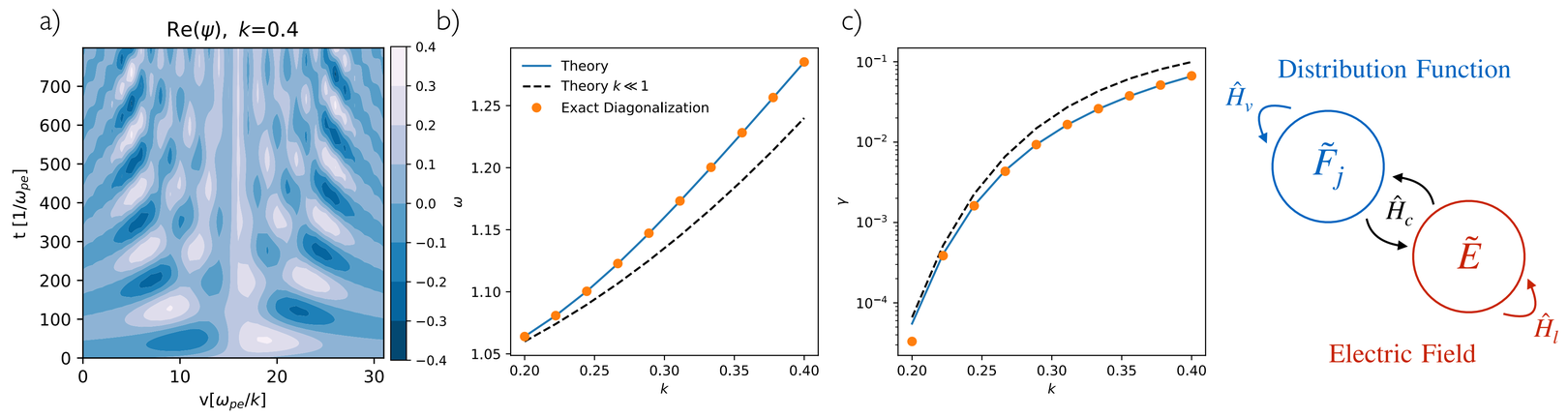


Figure 1: a) evolution of velocity distribution function, b) real frequency component from simulation and theory, c) growth rate of the instability

Non-Hermitian Hamiltonian - Two-stream Instability

In the cold two-stream instability, the dynamics can be encoded into a 4 × 4 matrix for the density n_1 and velocity v_1 perturbations of the beams.

General approach:

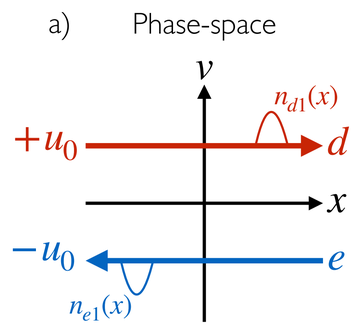
Write dynamical fluid/kinetic eqs $\partial_t X = F(X, Y, Z)$

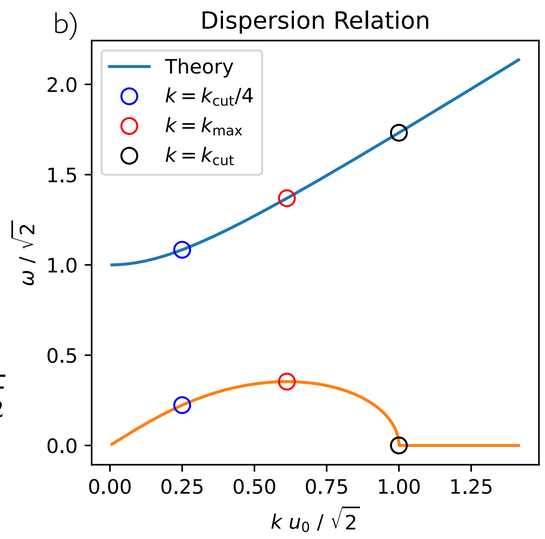
Linearize $X = \bar{X} + \tilde{X} = X_0 + X_1$

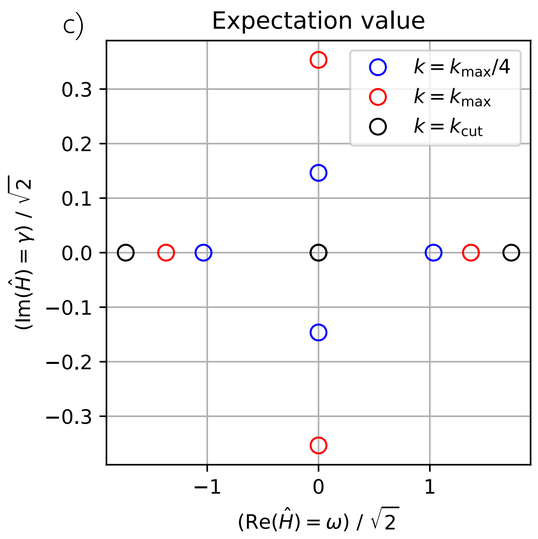
Use structure/symmetry to obtain a Schrödinger matrix eq

$\partial_t \psi = -i\hat{H}\psi, \quad \psi \equiv (X, Y, Z)^T$

Get real/complex eigenvalues of matrix $\hat{H}$







Dispersion relation: $\omega_r^2 = \omega_p^2 + k^2u_0^2 + \omega_p\sqrt{\omega_p^2 + 4k^2u_0^2}$, $\omega_i = \gamma(\xi) = \omega_p\sqrt{-(\xi^2+1) + \sqrt{1+4\xi^2}}$

Features: $\omega_{\min} = \sqrt{2}\omega_p,\ k_{\max} = \sqrt{3}\omega_p/(2u_0),\ k_{\text{cut}} = \sqrt{2}\omega_p/u_0$

Figure 2: a) sketch of phase-space for the fluid approximation, b) theoretical curves of dispersion relation with selected mode values, c) landscape where the local minima of $(\hat{H}^\dagger - E^*)(\hat{H} - E)$ are the complex frequency eigenvalues

Conclusions & Future work

First steps towards plasma dispersion relations with quantum computing
- Identified potential classes of simple problems to test approach on current quantum hardware

In the future:
- Optimize and implement the hybrid quantum algorithm, with state preparation, and error mitigation
- Study cost scaling with size of the Hamiltonian system (geometry, dimensionality and number of particle species)
- Emulate larger problems on classical HPC to study feasibility on future fault-tolerant quantum computers
- Manuscript in preparation [5]

References

[1] Choi K et al, PRL 127 (2021)
[2] Xie X D et al, FrontPhys 19 41202 (2024)
[3] Engel et al PRA (2019)
[4] Toyoizumi et al PRA (2024)
[5] ÓA, L. Gamiz, E. Koukoutsis, M.Vranic, in preparation (2027)

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