PhD Open Days

Solving stiff dark matter equations via Jacobian Normalization with Physics-Informed Neural Networks

PHD PROGRAMME IN PHYSICS (FCT SCHOLARSHIP)
JOSÉ ROCHA[1] (jose.r.rocha@tecnico.ulisboa.pt)

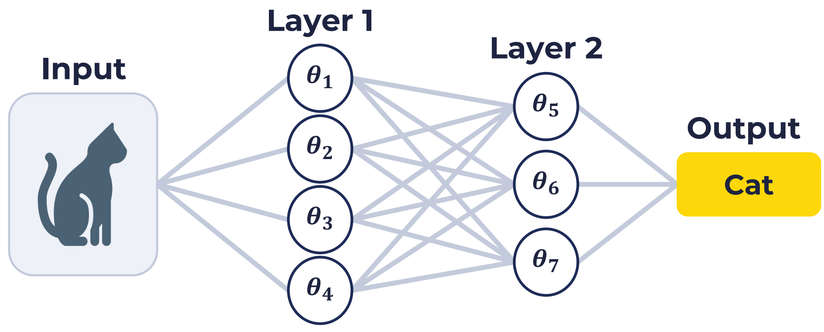
Read the full paper:
arXiv: 2602.21988

M. P. Bento CFTP
H. B. Câmara IEAP CTU
J. R. Rocha CFTP
J. F. Seabra CFTP

The problem

What is a neural network (NN)?

- It passes an input through layers of connected nodes, each carrying its own parameters θ
- Training tunes θ to shrink the gap between output and target
- Show it photos labeled “cat” or “not a cat” and it learns to spot cats it has never seen



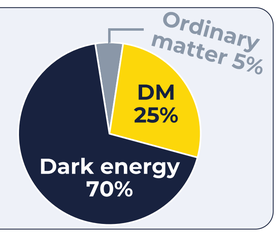
A NN trained to satisfy a differential equation instead of matching labels is called a Physics-Informed Neural Network (PINN)

Stiffness breaks PINNs

For systems with widely separated time scales, gradients from rapidly decaying modes dominate backpropagation, suppress the learning of slower modes and often **lead to convergence failure**

Dark matter (DM) obeys a stiff equation

Weakly interacting massive particles start in thermal equilibrium with the Standard Model plasma, then undergo **thermal freeze-out** as the Universe expands and cools, and their yield follows a stiff, nonlinear **Boltzmann equation**



The solution

Jacobian Normalization of the Loss Residual

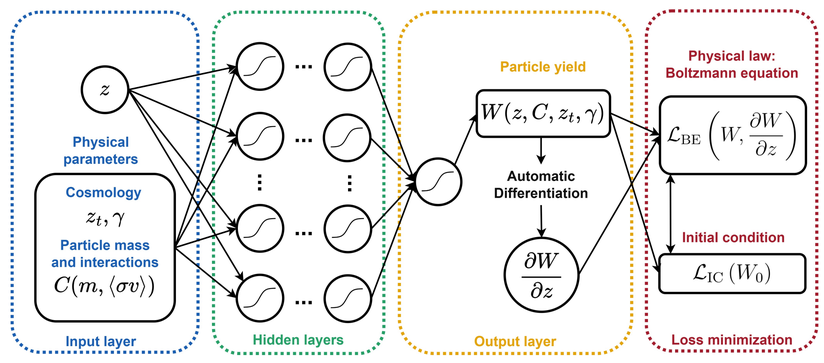
For $\frac{dy}{dx} = f(y,x)$, with **Jacobian** $J_y = \frac{\partial f}{\partial y}$, rescaling suppresses the growth of the loss **Hessian eigenvalues** $|\lambda_{max}| \sim \Theta(J_y^{2+a})$, restoring convergence:

$$\mathcal{L}_r = \frac{1}{m}\sum\left[\frac{\partial y}{\partial x} - f(y,x)\right]^2 \quad \Rightarrow \quad \frac{1}{m}\sum\frac{\left[\frac{\partial y}{\partial x} - f(y,x)\right]^2}{1 + J_y^2}$$

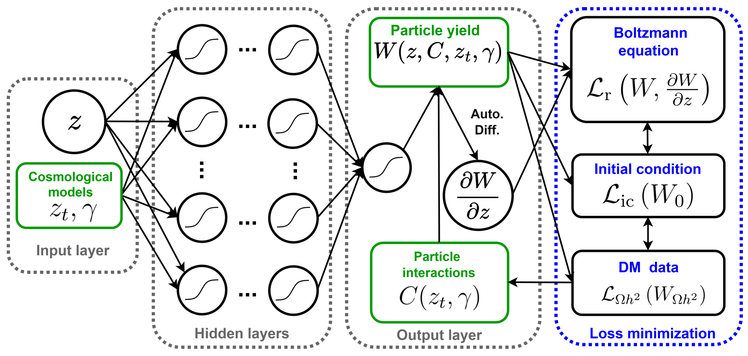
Normalizing the loss residuals with the Jacobian mitigates stiffness, without introducing extra hyperparameters



Forward PINN

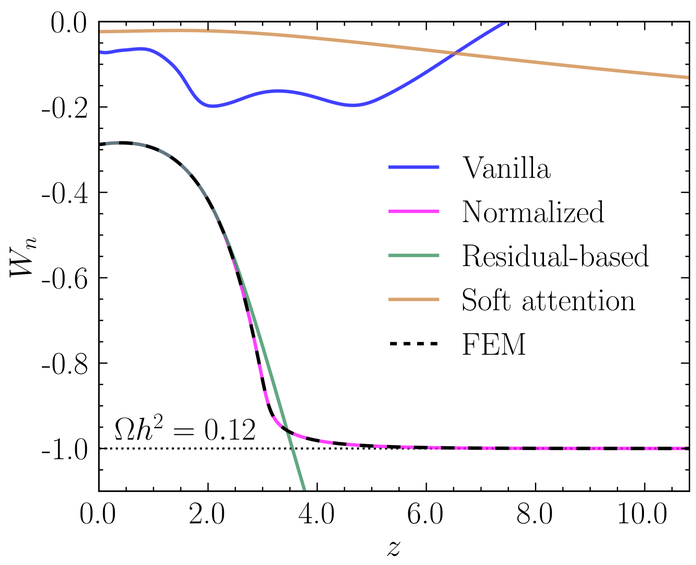


Inverse PINN



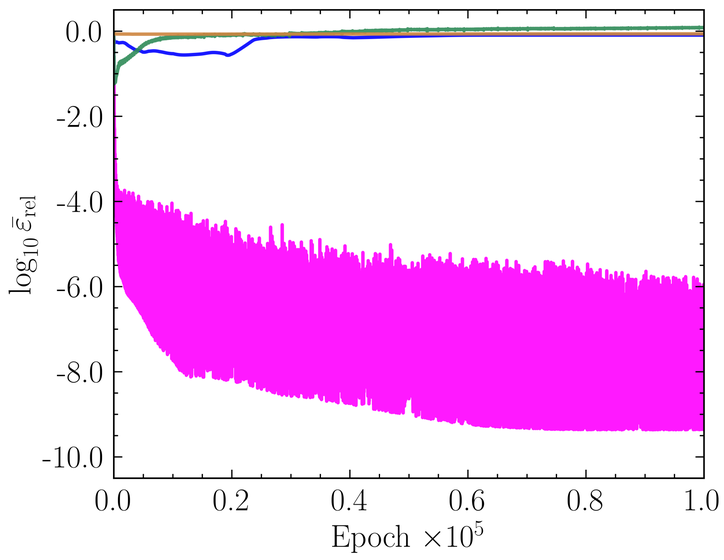
The results

Accuracy



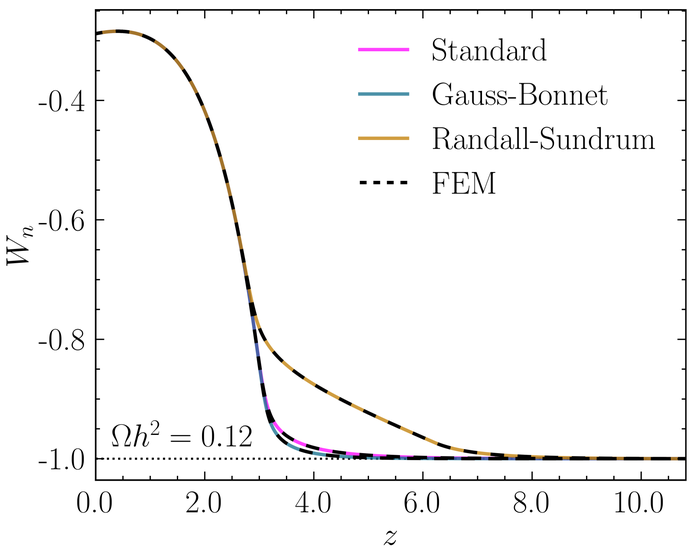
Only the Jacobian-normalized PINN reproduces the FEM result

Convergence



The mean relative error reaches $\bar{\varepsilon}_{rel} \sim 10^{-8}$ after only $\sim 10^4$ epochs

Inverse problem



From the relic density Ωh^2 alone, the **network inferred** the annihilation rate $\langle\sigma v\rangle$

Jacobian-based normalization is a simple way to make PINNs robust in stiff regimes, enabling more accurate and efficient modeling of dynamical systems at the interface of particle physics and cosmology

Acknowledgements:
Funding: UID/00777/2025, CERN/FIS-PAR/0019/2021
FCT PhD Grant: 2025.00959.BD

Supervisors: Filipe R. Joaquim[1] and José W. F. Valle[2]
[1] Centro de Física Teórica de Partículas, Instituto Superior Técnico, Universidade de Lisboa
[2] Institut de Física Corpuscular, CSIC/Universitat de València

phdopendays.tecnico.ulisboa.pt