Computational Mechanics with Peridynamics

Doctoral Program in Mechanical Engineering

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Introduction

Classical Theory of continuum mechanics

In classical continuum mechanics the equation of motion of a deformable solid, in Lagrangian form, is given in the reference configuration as

$$\rho_0 \ddot{\mathbf{u}} = \nabla \cdot \mathbf{P} + \mathbf{b}$$

where $\mathbf{P}$ is the first Piola-Kirchhoff stress tensor, $\mathbf{b}$ is the body force vector, ρ_0 is the undeformed mass density and $\mathbf{u}$ is the displacement field, and its second derivative is the acceleration field. The divergence of the stress tensor (which represents the equilibrium of internal forces) introduces restrictions to a given body deformation, since this deformation should be sufficiently smooth in order for the stress tensor to be differentiable.

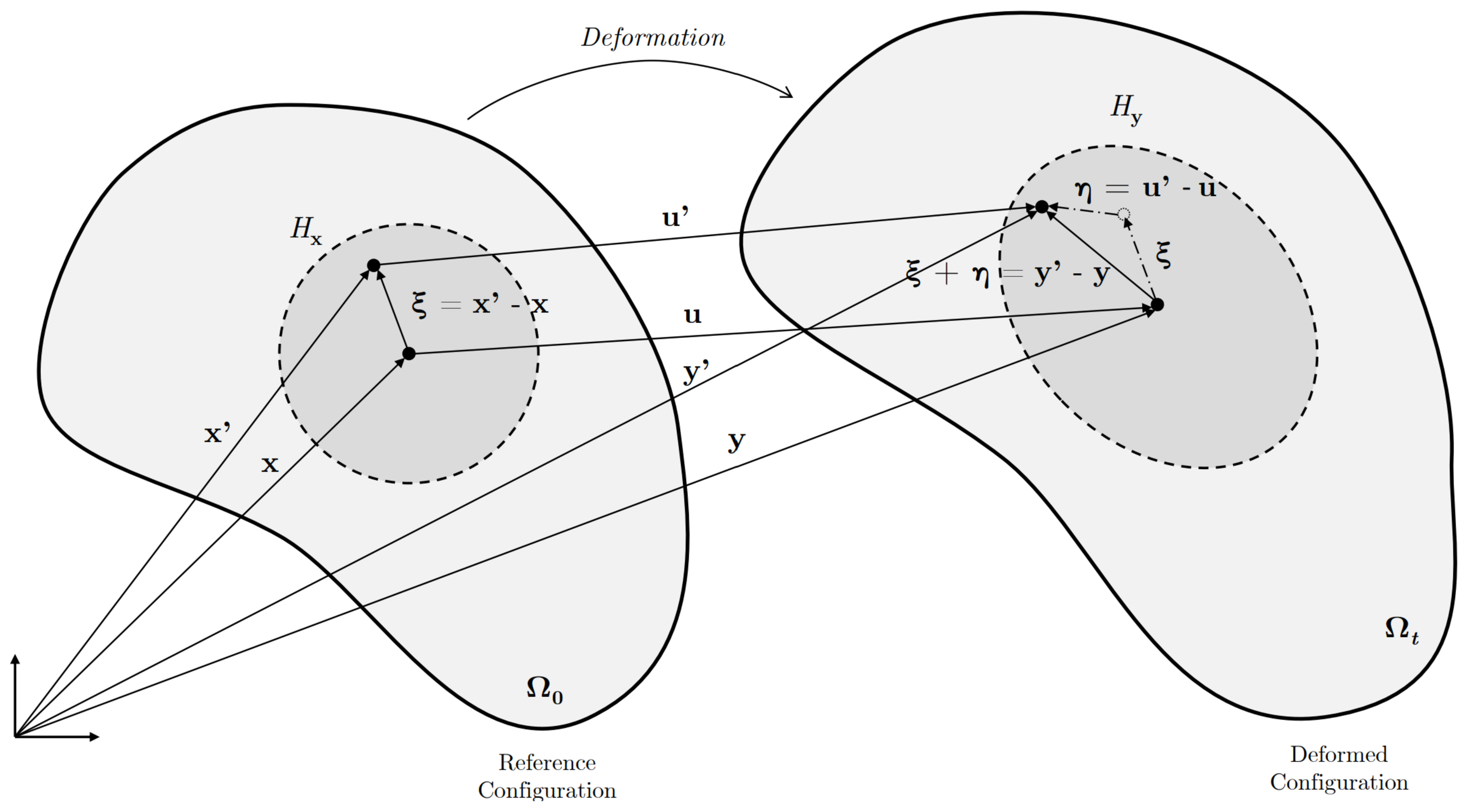
Peridynamic formulation

In order to overcome these difficulties, in peridynamics (PD) the equilibrium of internal forces is changed from the divergence of the stress tensor to an integral over a horizon of points

$$\rho_0 \ddot{\mathbf{u}}(\mathbf{x}) = \int_{H_{\mathbf{x}'}} \boldsymbol{f}(\mathbf{x'}, \mathbf{x}) dV' + \mathbf{b}(\mathbf{x})$$

In the above equation f is the bond force density, representing a bond interaction between point x and a neighbor x'. It also contains constitutive information.

Here, discontinuities such as cracks, which lead to singularities in the classical PDEs, do not produce any problems in the PD equations. Moreover, the removal of interactions between points makes it appropriate for conducting crack propagation analysis. Furthermore, the nonlocal character of the equations provides an adequate setting for the simulation of multiscale phenomena.



A peridynamic body subjected to deformation and its kinematics

Computational model

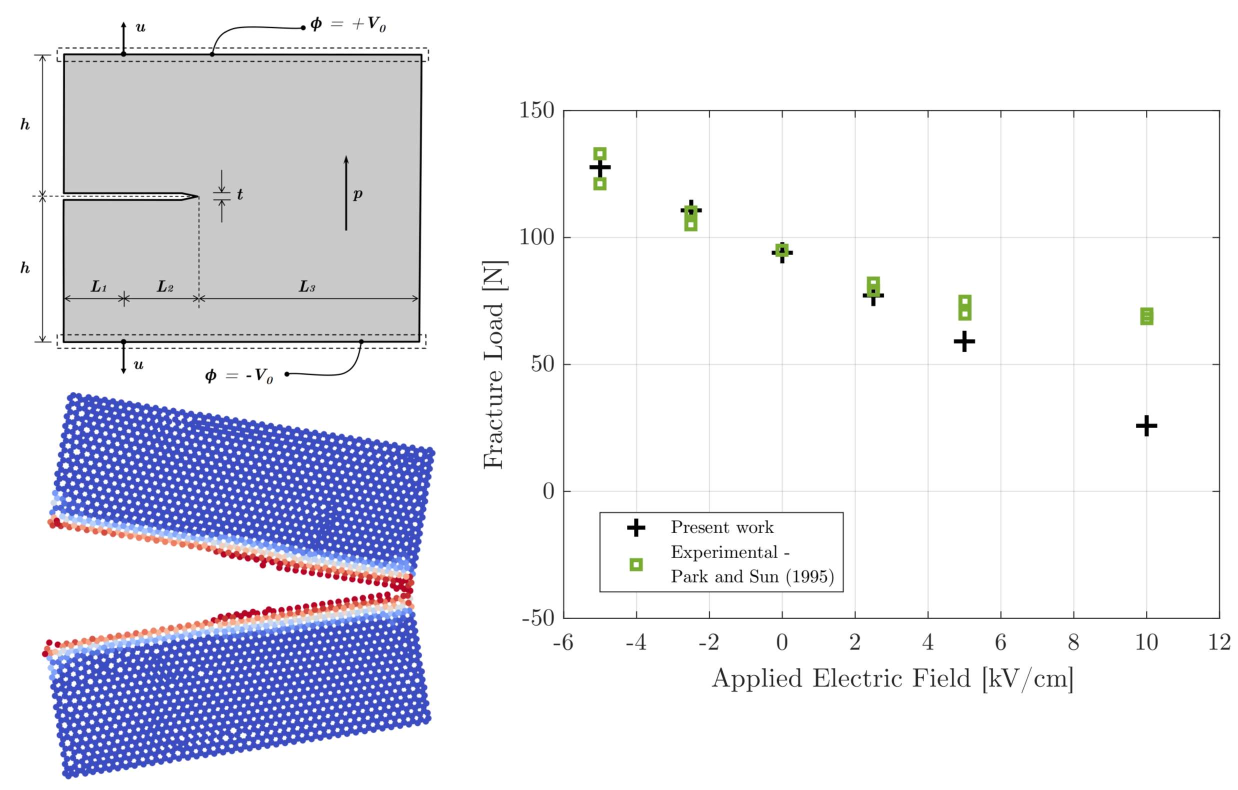
In peridynamics, one can simply discretize the equations with a straightforward meshless point collocation method, due to its simplicity. The domain is divided in a finite number of points, and to each point is attributed an area/volume

$$\rho_j \ddot{\mathbf{u}}^t = \sum_{n=1}^{m} \boldsymbol{f}(\boldsymbol{\eta}_{jn}, \boldsymbol{\xi}_{jn}) V_n + \mathbf{b}_j$$

Applications

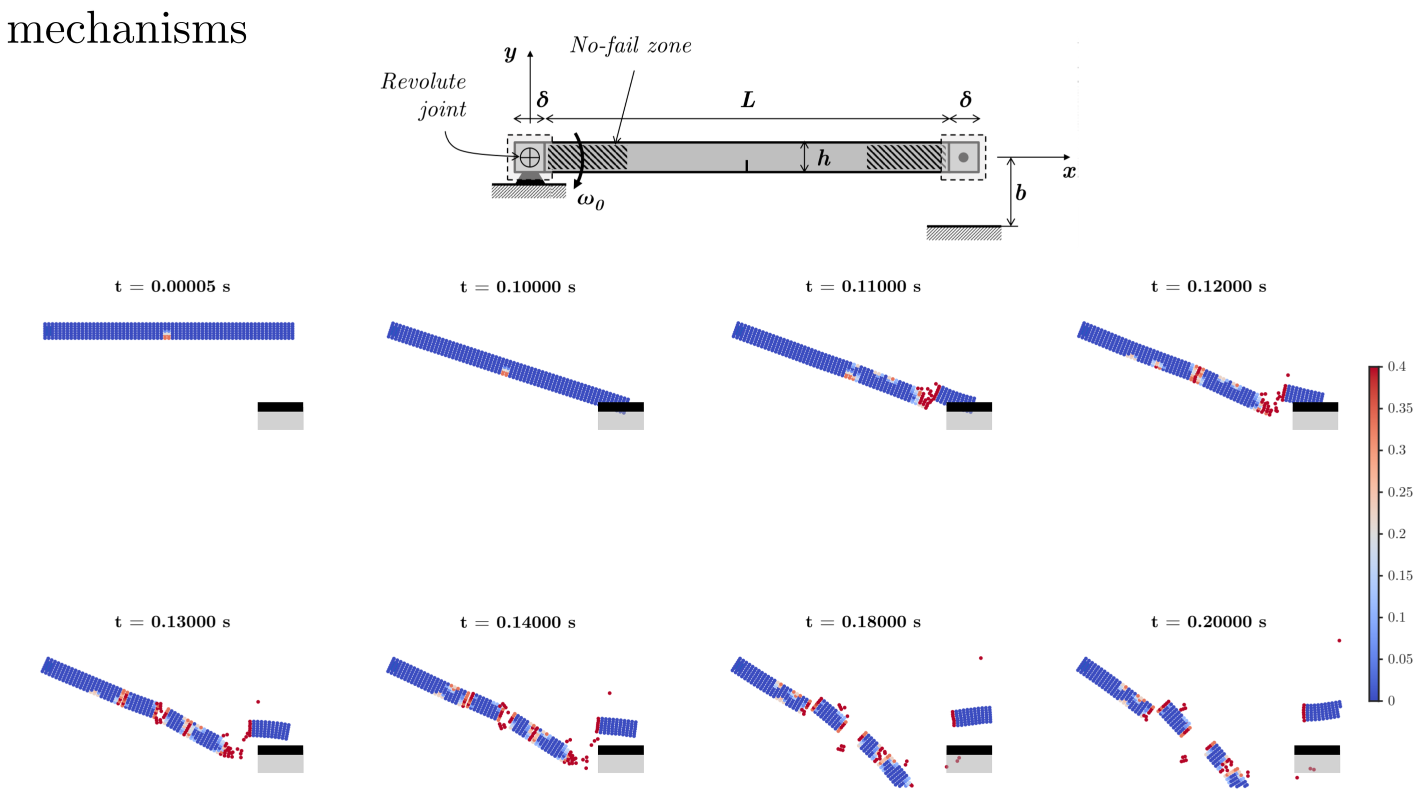
In the current work, different applications have been developed to demonstrate the capabilities and advantages of peridynamics in computational mechanics

- Firstly, we present results of an electromechanical model for the simulation of fracture in piezoelectric solids



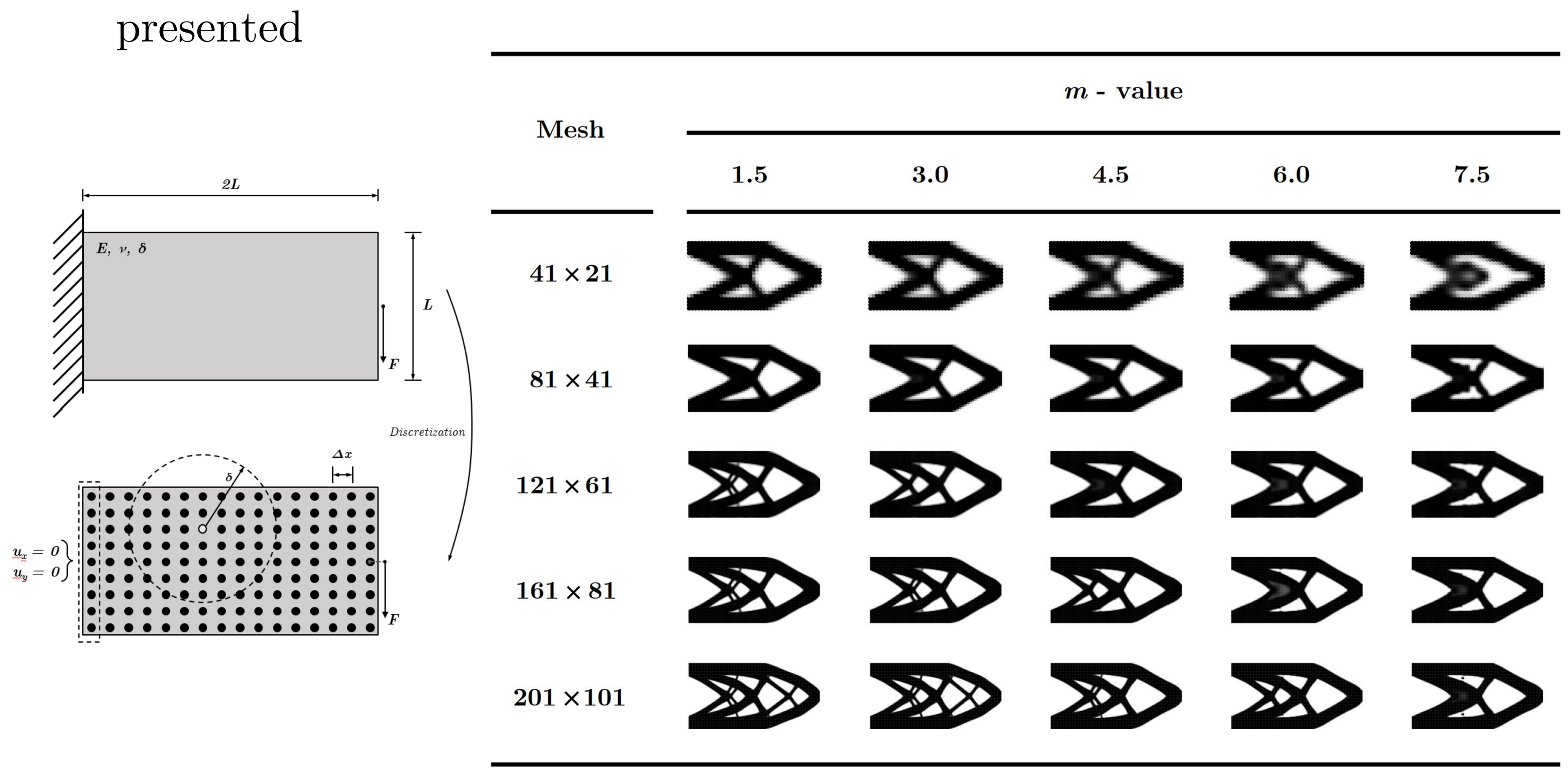
Piezoelectric tension test: a notched specimen is subjected to traction loading, leading to crack propagation. The fracture loads obtained in the simulations show good agreement with experimental results, except at high applied electric fields.

- Secondly, we present results from an integrated multibody dynamics with fracturable flexible bodies framework, for simulation of multibody mechanisms



Pendulum subjected to impact: A pendulum with initial velocity impacts an obstacle leading to catastrophic failure.

- Finally, application of peridynamics to topology optimization designs is presented



Topology optimization of a cantilever beam with centered load: results demonstrate mesh independency due to the nonlocal character of the peridynamic formulation. The horizon, the intrinsic peridynamic length scale, restricts the features and details of the optimal designs.