



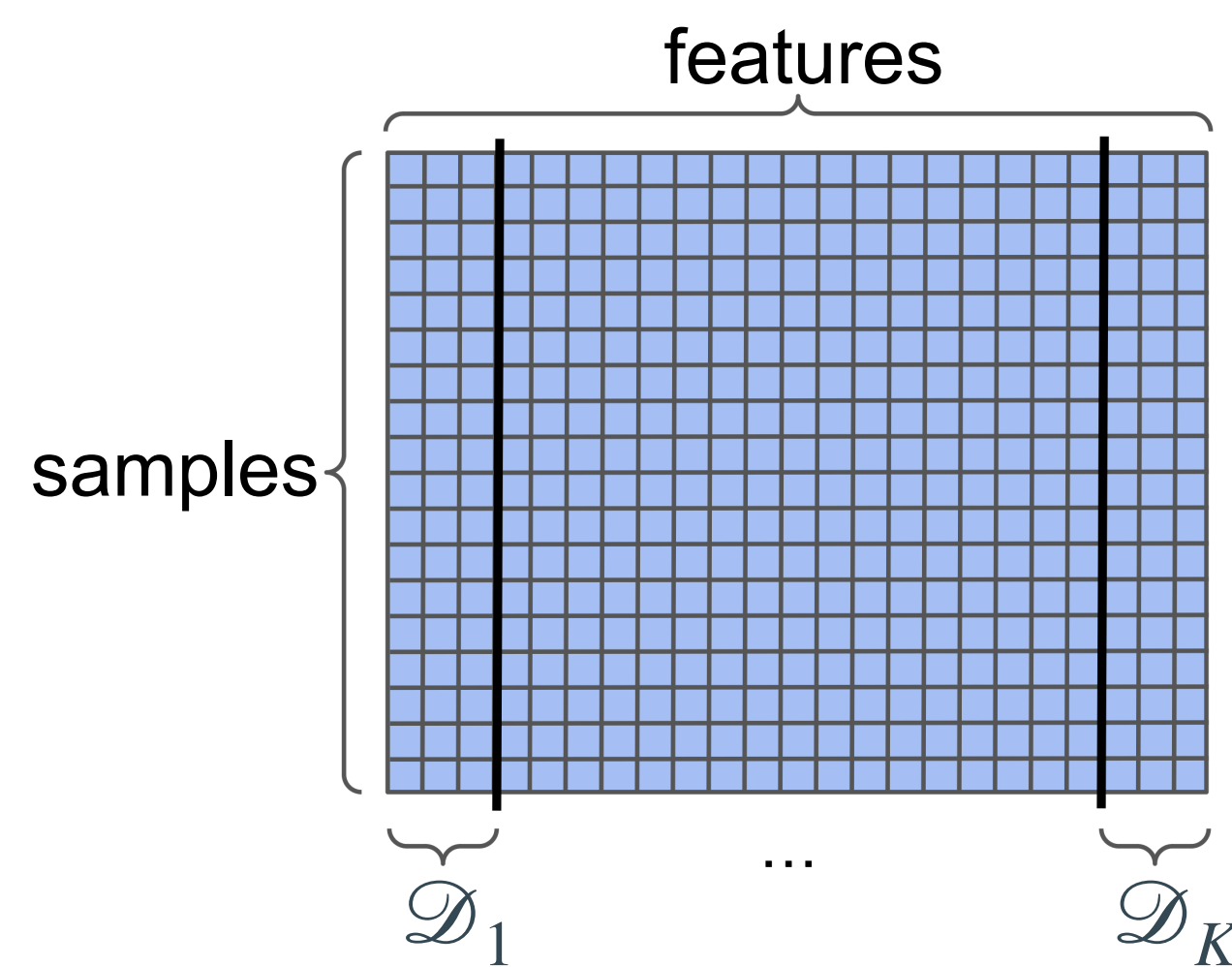
Error Feedback Compressed Vertical Federated Learning

Carnegie Mellon Portugal Double Degree Program

Pedro Valdeira, pvaldeira@cmu.edu

Background on Vertical Federated Learning

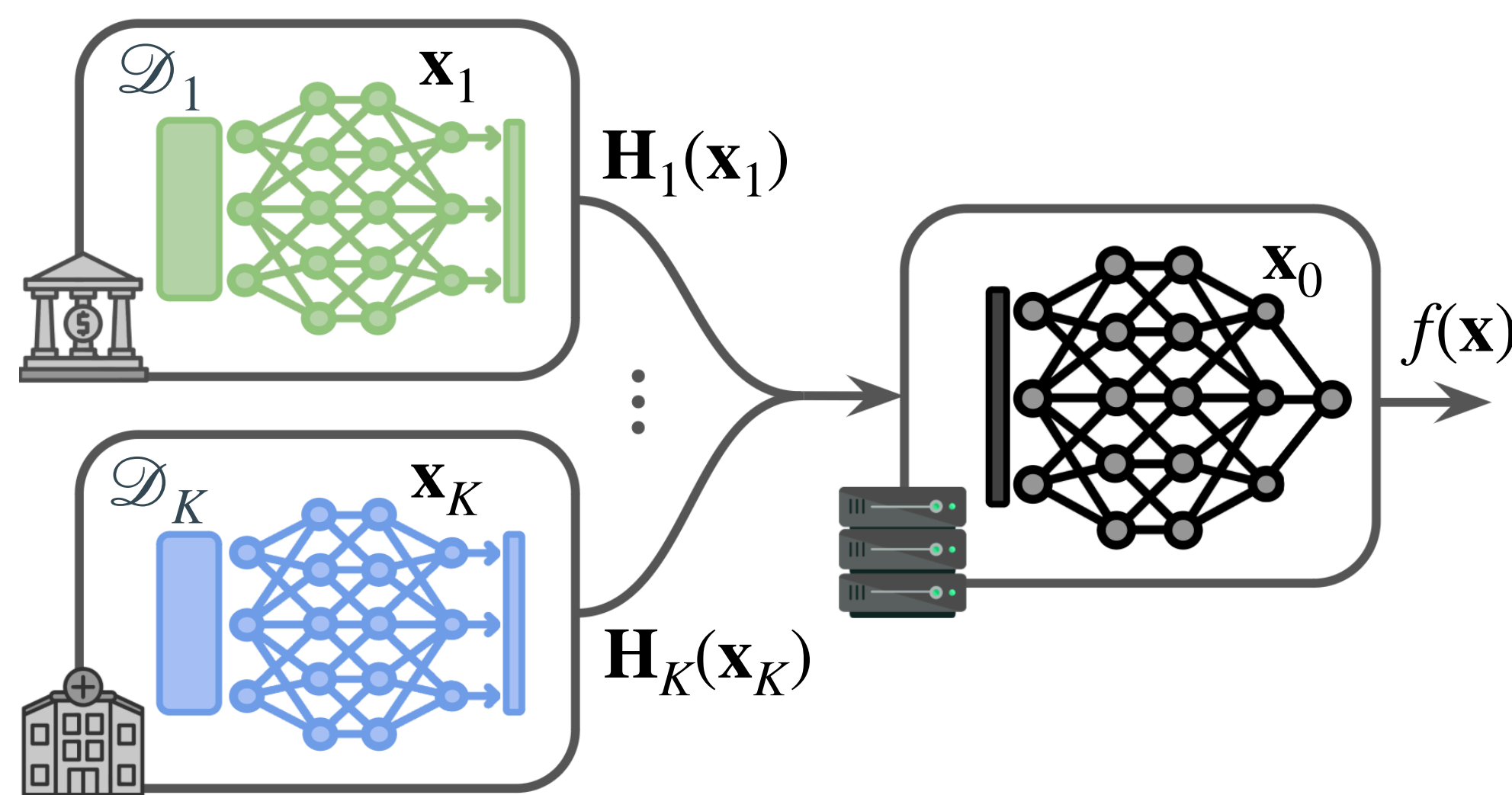
Dataset \mathcal{D} partitioned across K clients,



who want to collaborate to learn the parameters $\mathbf{x} := (\mathbf{x}_0, \mathbf{x}_1, \dots, \mathbf{x}_K)$ of a model:

$$f(\mathbf{x}) := \phi(\mathbf{x}_0, \mathbf{H}_1(\mathbf{x}_1), \dots, \mathbf{H}_K(\mathbf{x}_K)),$$

where $\mathbf{H}_k(\mathbf{x}_k) := \mathbf{H}_k(\mathbf{x}_k; \mathcal{D}_k)$, without sharing local data.



Communication is often the bottleneck, so communication efficiency is a major challenge.

Prior art

Algorithm : Compressed Vertical Federated Learning [1]

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Input: initial point  $\mathbf{x}^0$ , step-size  $\eta$ 
1 for  $t = 0, \dots, T-1$  do
2   Server samples  $\mathcal{B}^t \subseteq [N]$  and sends  $\mathcal{B}^t$  and  $\mathbf{x}_0^t$  to all clients
3   for  $k \in [K]_0$  in parallel do
4     Compute  $\mathbf{x}_k^{t+1}$  at  $k$  using coordinate descent
5     if  $k > 0$  then
6       Client  $k$  computes and sends  $\mathcal{C}(\mathbf{H}_{k\mathcal{B}^t}(\mathbf{x}_k^{t+1}))$  to the server
    
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We say $\mathcal{C}: \mathbb{R}^d \rightarrow \mathbb{R}^d$ is a (biased) compressor if there exists $\alpha \in (0, 1]$ such that

$$\mathbb{E} \|\mathcal{C}(\mathbf{x}) - \mathbf{x}\|^2 \leq (1 - \alpha) \|\mathbf{x}\|^2, \quad \forall \mathbf{x} \in \mathbb{R}^d.$$

This includes top- k sparsification, quantization,...

Problem of direct compression: compression error does not vanish unless $\alpha \rightarrow 1$, requiring a vanishing stepsize to converge at a $\mathcal{O}(1/\sqrt{T})$ rate.

EFVFL (our method)

Algorithm : Error Feedback Compressed Vertical Federated Learning

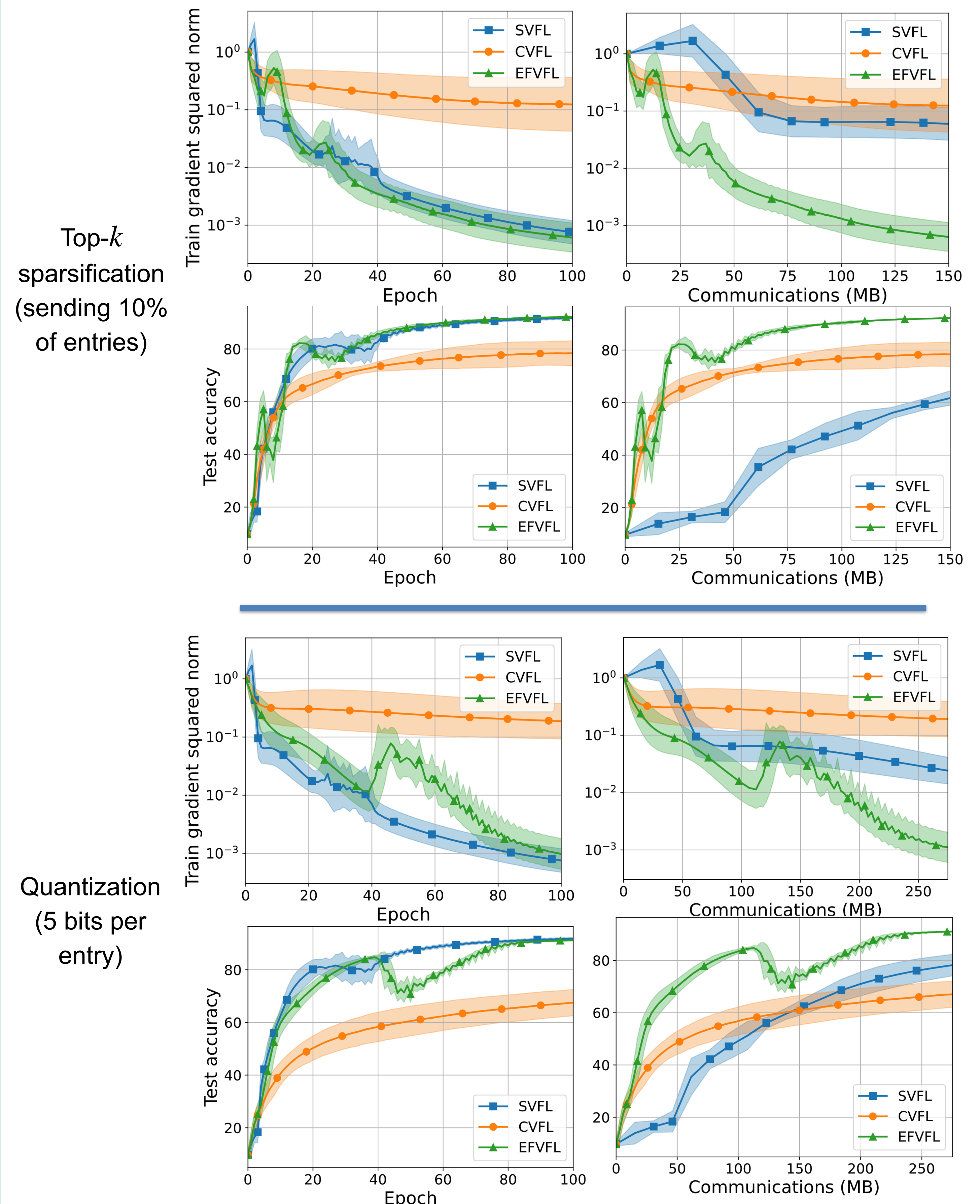
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Input: initial point  $\mathbf{x}^0$ , step-size  $\eta$ , initial estimates  $\{\mathbf{G}_k^0 = \mathcal{C}(\mathbf{H}_k(\mathbf{x}_k^0))\}$ 
1 for  $t = 0, \dots, T-1$  do
2   Server samples  $\mathcal{B}^t \subseteq [N]$  and sends  $\mathcal{B}^t$ ,  $\{\mathbf{G}_{k\mathcal{B}^t}^t\}$ , and  $\mathbf{x}_0^t$  to all clients
3   for  $k \in [K]_0$  in parallel do
4     Compute  $\mathbf{x}_k^{t+1}$  at  $k$  using a coordinate descent step
5     if  $k > 0$  then
6       Client  $k$  computes  $\mathbf{C}_k^t = \mathcal{C}(\mathbf{H}_{k\mathcal{B}^t}(\mathbf{x}_k^{t+1}) - \mathbf{G}_{k\mathcal{B}^t}^t)$  and sends it to the server
7       Update  $\mathbf{G}_{k\mathcal{B}^t}^{t+1} = \mathbf{G}_{k\mathcal{B}^t}^t + \mathbf{C}_k^t$  and  $\mathbf{G}_{ki}^{t+1} = \mathbf{G}_{ki}^t$  for  $i \notin \mathcal{B}^t$  at client  $k$  and at the server
    
```

Convergence guarantees:

- If f and ϕ are L -smooth and \mathbf{H}_k have a bounded gradient norm, EFVFL converges at a $\mathcal{O}(1/T)$ rate;
- If we further assume the PL condition, we have linear convergence.

Experiments (shallow network to classify MNIST)



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References: [1] Castiglia, Timothy J., et al. "Compressed-VFL: Communication-efficient learning with vertically partitioned data." ICML 2022.