# PhD Open Days

# Simpler Quantum-Classical Interpolation with New Tools

Hybrid Quantum-Classical High-Performance Computation

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## The Problem

### Quantum Computing

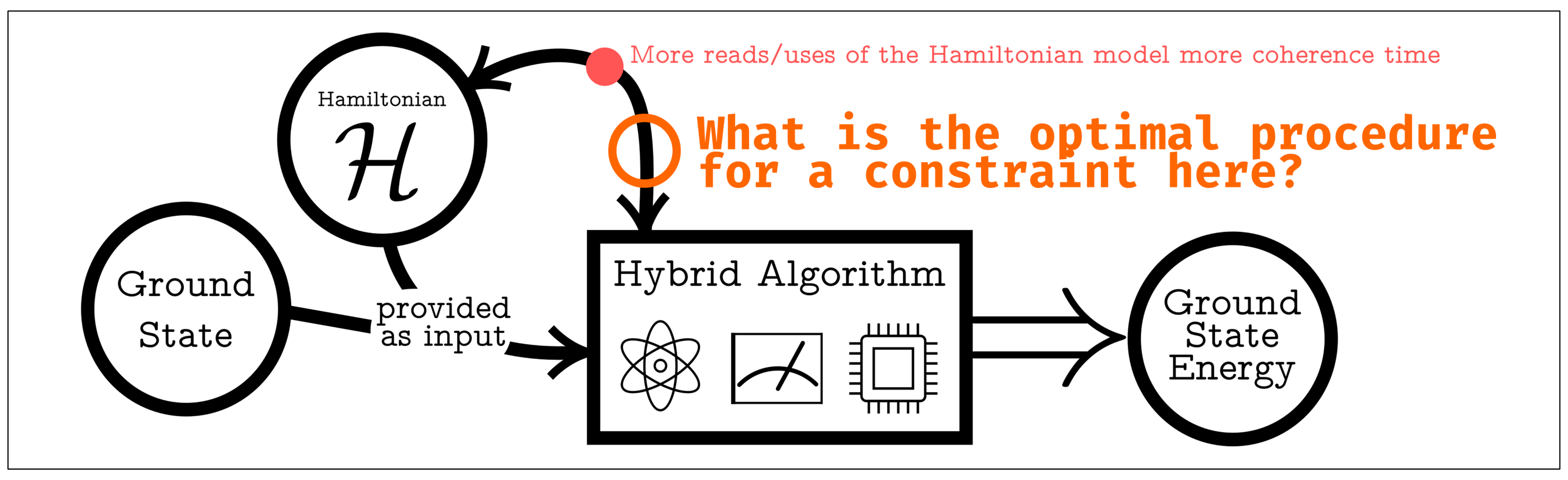
Are there problems that are solved faster with this model (as opposed to the classical model)?
**Yes!** (Provided some long-standing conjectures are true.)

We seek similar statements for the **hybrid** situation:
when quantum computing is interspersed with classical computing, and obtain an advantage for any limitation on the coherence time of the quantum computer.

### Phase Estimation Problem

The ability to solve it coherently underlies Shor's Factoring Algorithm [1,2].
(This is one such problem where there probably is *exponential* quantum advantage.)

One should distinguish the Phase Estimation Sampling algorithm (fully-coherent) from the **Phase Estimation** algorithm (allows for intermediate measurements).
The Phase Estimation algorithm provides a quadratic advantage.

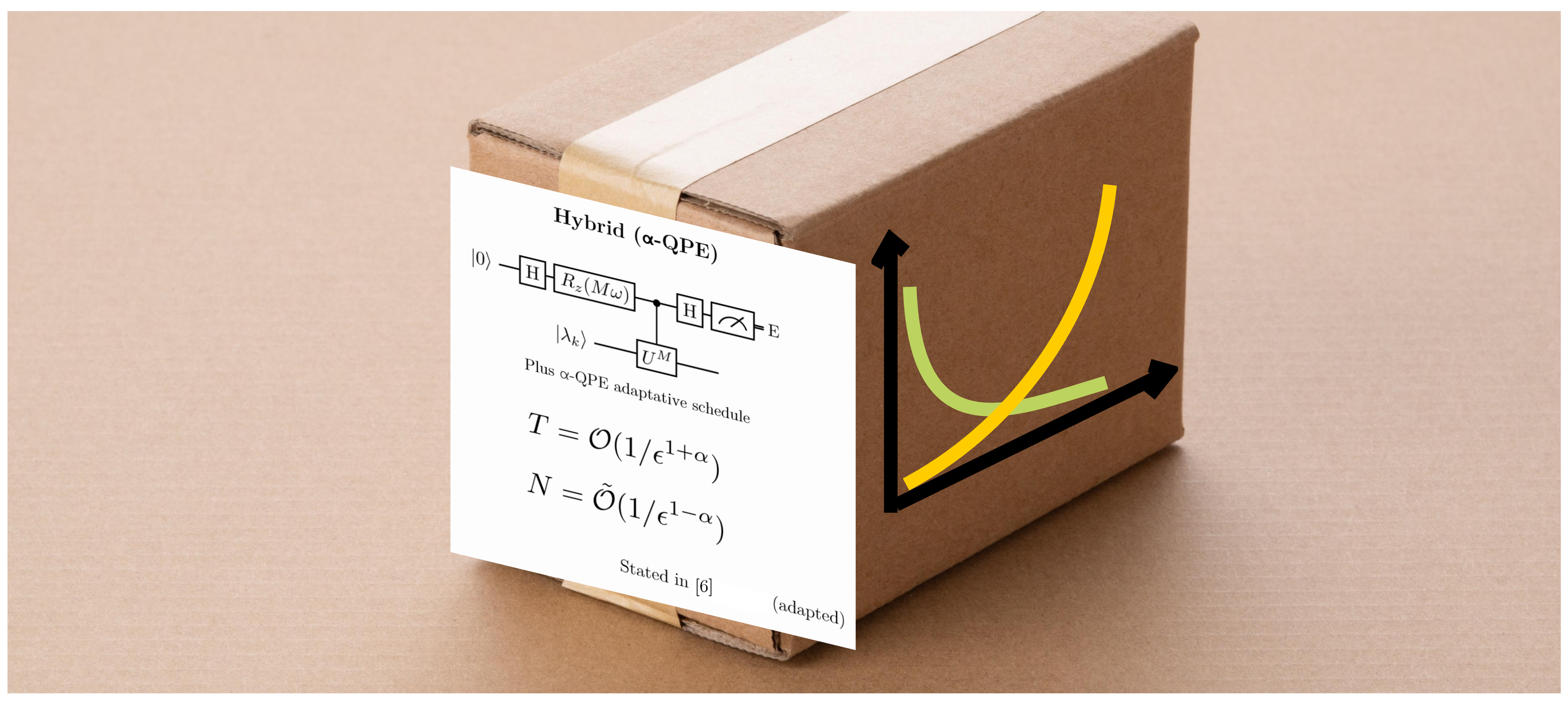



SUBTITLE: Phase Estimation is a particular case of Eigenvalue Estimation. How to best solve it with a limited quantum computer, aided by a classical one?

### Known Solutions With Varying Amounts of Coherence

Kitaev [3], Svore *et al.* [4] and Weibe and Granade [5] progressively introduced hybrid quantum-classical algorithms to solve Phase Estimation

Wang *et al.* showed [6] that **more coherent calls to U meant less overall calls to U.**
(Equivalently, more coherent calls mean less measurements overall.)




SUBTITLE: Wang *et al.*'s work show exactly how one can trade off the total number of calls to an oracle (T) by number of measurements (N), but their construction relies on a historical derivation and does not hint on how to generalize the construction.

## A New Approach

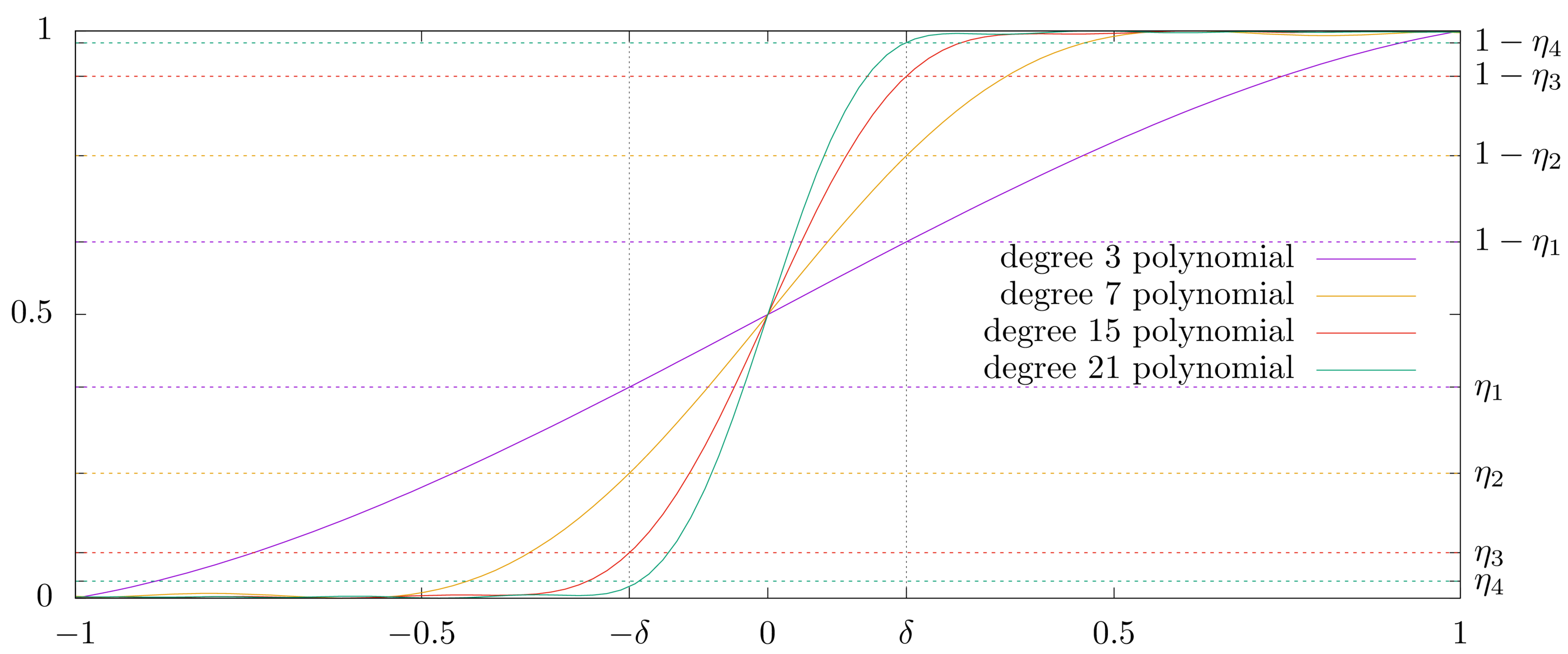
### Quantum Singular Value Transformations (QSVTs)

- Gilyén *et al.* introduced QSVT in 2019 [7].
- Many quantum computations can be thought of as applications of a polynomial to the input oracle.
- Calling the oracle $2k$ times allows for a $k$-degree polynomial.
- Implementing a $k$-degree polynomial is done by setting $k+1$ rotation angles, which can be obtained prescriptively.

### Our Contribution [8]

1. Noting that solving the **Eigenvalue Estimation** (EE) problem is sufficient to solve Phase Estimation.
   (This means statements on hybrid solutions to EE are more general.)
2. Noting that solving the decision version of the problem is enough.
   (Instead of answering *what is the eigenvalue*, ask *is the eigenvalue less than or greater than some value*)
3. Noting that there is a simple formulation to answer the decision problem by applying a polynomial to the Hamiltonian. (Use a step function.)
4. Noting that hybrid approaches translate simply to coarser approximations to the relevant polynomial, compensated by more measurements.

Setting δ and η to particular values recovers the known literature results and coherence-measurement trade-offs.




SUBTITLE: An increasing number of coherent calls to a block-encoding of the input Hamiltonian allows the implementation of a better approximation of the step function: O(1/δ log(1/η)) calls to the oracle gives a step function approximation within the δ, η tolerances shown in the figure. When the approximation is poor, more measurements must be taken to compensate.

### Takeaway

This re-derivation is more general in two senses:

1. It **establishes a family of hybrid algorithms for Eigenvalue Estimation**, which is a more general problem than Phase Estimation;
2. It is "from first principles," with more **potential to be applied to other algorithms for similar results**.

### Future Goals

- Apply this technique to obtain hybrid algorithm interpolations for other algorithms.
- Understand hybridization limitations in light of this formulation.

### References

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[2] Nielsen, M., & Chuang, I. (2010). Quantum Computation and Quantum Information: 10th Anniversary Edition. Cambridge University Press. Pp. 226-234.
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[4] Svore, K., Hastings, M., & Freedman, M. (2014). Faster Phase Estimation. Quantum Info. Comput., 14(3–4), 306–328.
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[6] D. Wang, O. Higgott, and S. Brierley, Accelerated variational quantum eigensolver, Physical Review Letters **122**, 10.1103/physrevlett.122.140504 (2019).
[7] A. Gilyén, Y. Su, G. H. Low, and N. Wiebe, Quantum singular value transformation and beyond: exponential improvements for quantum matrix arithmetics, in Proceedings of the 51st Annual ACM SIGACT Symposium on Theory of Computing (ACM, 2019).
[8] Magano, D., Murça, M.. (2022). Simplifying a classical-quantum algorithm interpolation with quantum singular value transformations.