PhD **Open Days**

2nd edition!

16 - 17 MAY / SALÃO NOBRE

Fluid Plasma Model with Kinetic Coulomb Collisions

APPLAuSE

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Boltzmann Equation

Distribution Function

Stores all information about the collective behaviour of the plasma

$$\frac{\partial f_a}{\partial t} + \dot{\mathbf{x}} \cdot \nabla_{\mathbf{x}} f_a + \dot{\mathbf{v}} \cdot \nabla_{\mathbf{v}} f_a = \sum_b C(f_a, f_b)$$

Collision Operator

Models collisions between different charged species

$$C(f_a, f_b) = \frac{\ln \Lambda}{8\pi} \left(\frac{e_a e_b}{m_a \epsilon_0}\right)^2 \frac{\partial}{\partial v_i} \left[\frac{\partial^2 G_b}{\partial v_i \partial v_j} \frac{\partial f_a}{\partial v_j} - \frac{m_a}{m_b} \frac{\partial H_b}{\partial v_i} f_a\right]$$

$$H_b = 2 \int \frac{f_b(\mathbf{v}')}{|\mathbf{v} - \mathbf{v}'|} d\mathbf{v}', \quad G_b = \int f_b(\mathbf{v}') |\mathbf{v} - \mathbf{v}'| d\mathbf{v}'$$

Guiding Center

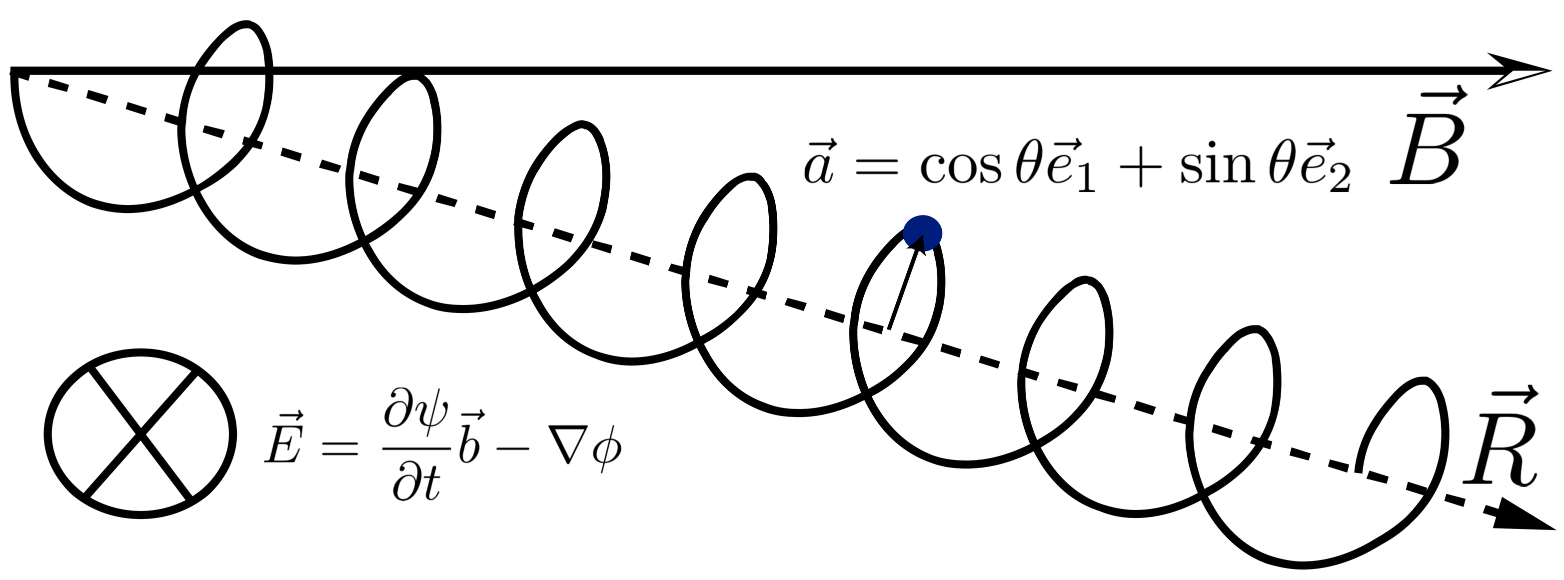
Coordinates

$$\mathbf{x} = \mathbf{R} + \epsilon \rho \mathbf{a}, \ \ \mathbf{v} = v_\parallel \mathbf{b} + \epsilon \rho \dot{\mathbf{a}}$$

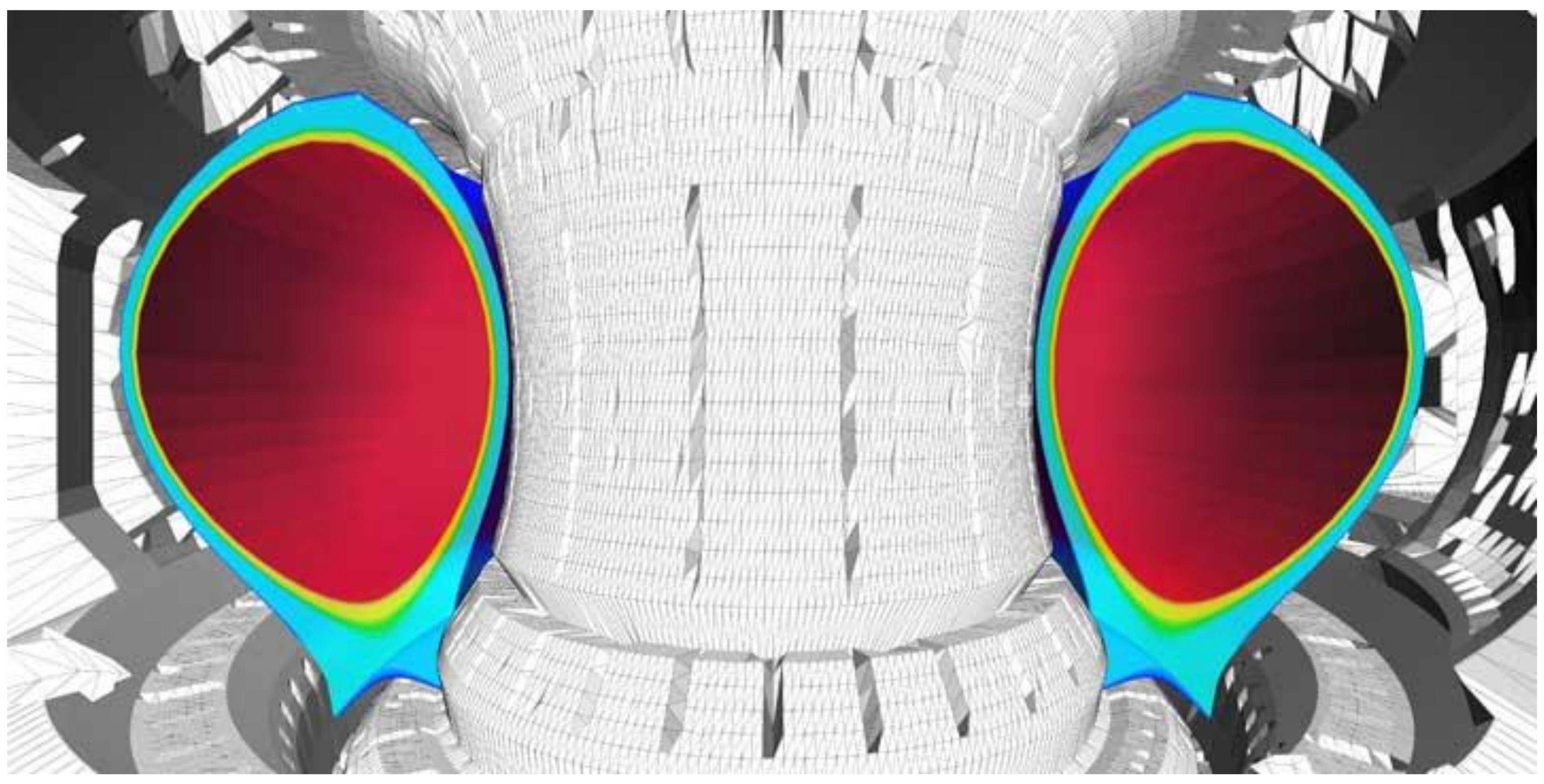
Drifts

$$\dot{\mathbf{R}} = v_\parallel \mathbf{b} + v_\parallel^2 \mathbf{u}_k + \mu \mathbf{u}_{\nabla B} + \mathbf{u}_E + \mathbf{u}_p$$

$$m \dot{v}_\parallel = \frac{\partial \psi}{\partial t} - \nabla_\parallel \phi - \nabla_\parallel \mu B + \frac{m \nabla_\parallel u_E^2}{2}$$



Towards Tokamak SOL Plasma Turbulence Understanding



Ji & Held, Phys. Plasmas 16, 2009

$$f = \sum_{l,k} \frac{f_M}{\sigma_k^l} f_M L_k^{l+1/2}(s^2) \mathbf{P}^l(\mathbf{s}) \cdot \mathbf{M}^{lk}$$

For the first time, we can integrate the collision operator explicitly

$$\int d\mathbf{v} C(f_a, f_b) \mathbf{P}^j(\mathbf{s}) L_p^{j+1/2}(s^2) = \sum_{lkm} \sum_{nqr} L_{km}^l L_{qr}^n C_{ab}^{jp,lk,nq} \overline{\mathbf{M}_a^{lk} \ .^{i+u} \ \mathbf{M}_b^{nq}}$$

Can we do the same in guiding center coordinates?

Gyro Averaged System

$$\frac{\partial (B \langle f \rangle)}{\partial t} + \nabla \cdot (\dot{\mathbf{R}} B \langle f \rangle) + \frac{\partial (\dot{v}_\parallel B \langle f \rangle)}{\partial v_\parallel} = B \langle C(f) \rangle$$

$$N_a^{lk} = \int H_l(s_\parallel) L_k(s_\perp^2) B \langle f_a \rangle \, d\mu dv_\parallel$$

$$\langle f \rangle = \sum_{pj} \frac{f_M}{\psi^p} N^{pj} H_p(s_\parallel) L_j(s_\perp^2)$$

Integrating the gyro averaged Boltzmann equation leads to the first drift-kinetic full Coulomb collision moment hierarchy

$$\frac{\partial (B N_a^{lk})}{\partial t} + \nabla \cdot \left\{ B \left[N_a^{lk} (\mathbf{u}_E + \mathbf{u}_p) + \left(\frac{N_a^{l+1k}}{4(l+1)} + 2l^2 N_a^{l-1k} \right) \mathbf{b} + \left(\frac{N_a^{l+2k}}{16(l+2)(l+1)} + l(l+1/2) N_a^{lk} \right.\right.\right.$$
$$\left.\left.\left. + 4l^2(l-1)^2 N_a^{l-2k} \right) \mathbf{u}_k + \frac{m_a}{2B} \left[k(2N_a^{lk} - N_a^{lk-1} - N_a^{lk+1}) + (N_a^{lk} - N_a^{lk+1}) \right] \mathbf{u}_{\nabla B} \right] \right\}$$
$$+ 2 m_a l^2 \left[N_a^{l-1k} E_\parallel^* + k(2N_a^{lk} - N_a^{lk-1} - N_a^{lk+1}) + (N_a^{lk} - N_a^{lk+1}) \right] \nabla_\parallel B = B \sum_{p,j,n,q} \mathcal{C}_{ab,lk}^{pj,nq} \mathcal{N}_a^{pj} \mathcal{N}_b^{nq}$$

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