PhD Open Days

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PIC simulations of the MagnetoRotational instability

APPLAuSE – Advanced Program in Plasma Science and Engineering

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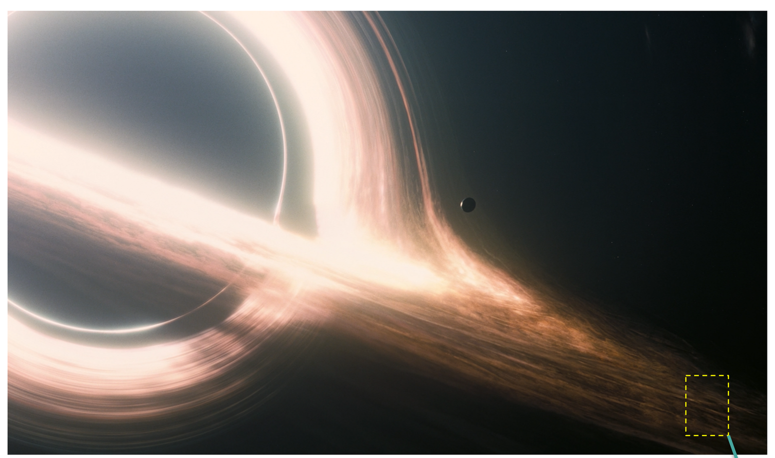
The Magnetorotational Instability



The Magnetorotational instability (MRI) is a crucial mechanism of angular momentum transport in astrophysical accretion disks.

The MRI has been widely studied using Magneto HydroDinamic models [1] and simulations.

Limit of weak magnetic field [2]

$$\frac{\omega_{ci}}{\alpha} << 1$$

Simulation box

$$L << r_0$$

The plasma is macroscopically collisionless and MHD breaks down.

We want to analyze this limit introducing a shearing co-moving frame equations in our PIC code OSIRIS 3.0 [3]. In this work we will present the analysis of the numerical stability of the 1D new Maxwell's equations.

Shearing co-moving frame equations

Non relativistic limit $$v_0 = r_0 \times \alpha << c$$

The Maxwell's equations and the motion equation become [4]:

$$\frac{\partial B(r,t)}{\partial t} = -\nabla \times E(r,t) - \frac{3}{2}\alpha B_y \hat{z}$$

$$\frac{\partial E(r,t)}{\partial t} = \nabla \times B(r,t) - \frac{3}{2}\alpha E_y \hat{z} - 4\pi J$$

$$\frac{dp}{dt} = 2\alpha p_z \hat{y} - \frac{1}{2}\alpha p_y \hat{z} + q\left(E + \frac{u}{c} \times B\right)$$

Numerical stability of 1D EM solver

We modified the classical Yee scheme for EM fields with a second order Runge-Kutta algorithm (RK2) in time. In this way, we are able to obtain the analytical solution for dispersion relation of wave propagation in this frame, numerically corrected up to 4th order in both time and space.

$$\omega^4 - 2k^2\omega^2 + k^4 - \frac{9}{4}\alpha^2 k^2 + O[dt^5, dx^5]$$

In figure (1) we can observe that the signal becomes numerical instable for high k and this instability will dominate to the signal, spoiling the propagation of the physical wave.

FFT vs time for $\alpha = 0.1$ $[1/\omega_{pe}]$ for a wake of $k=5$ $[\omega_{pe}/c]$

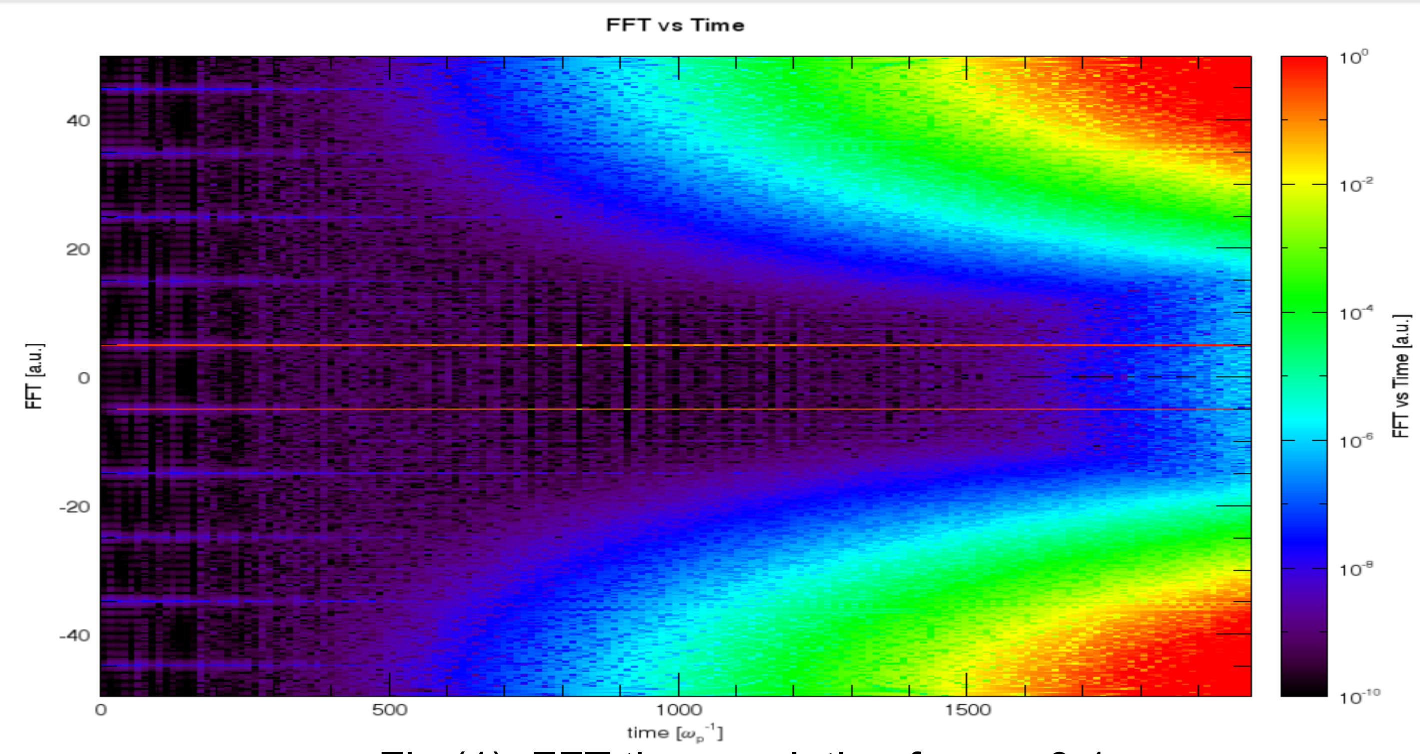


Fig (1): FFT time evolution for $\alpha = 0.1$

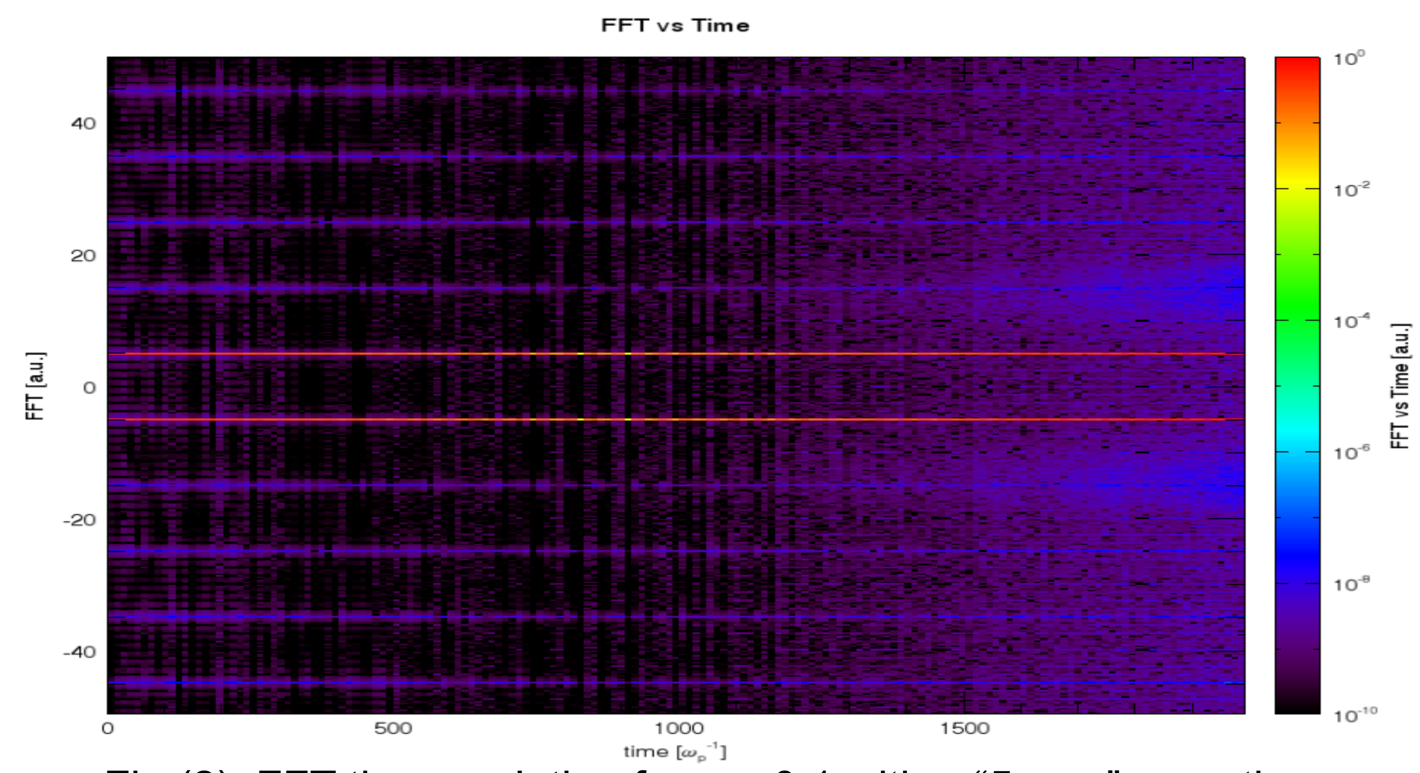


Fig (2): FFT time evolution for $\alpha = 0.1$ with a "5pass" smooth

How suppress numerical instability?

Use an extremely high time resolution → Not convenient: evolution of the system for long time (on the order of 5 x 10^4 $[1/\omega_{pe}]$)

Use a low-pass filtering → "5pass" binomial smoothing for high wavenumbers (figure 2)

Conclusions

We presented the shearing co-moving frame set of equations, which will be useful to study the evolution of MRI using our PIC code OSIRIS 3.0. In particular, we analyzed the numerical stability of the modified Maxwell's equations, solved using a combination of Yee and RK2 scheme. We shown that the equations used generate a numerical instability for high wavenumbers, that can be suppressed using a "5pass" smoothing in OSIRIS 3.0

References

[1] S. A. Balbus, J. F. Hawley, The Astrophys. Jour. 376, 214 (1991)
[2] J. Krolik, E. Zweibel, The Astrophys. Jour. 644, 651 (2006)
[3] Fonseca et al., Lect. Notes Comput. Sci. 2331, 342 (2002)
[4] M. A. Riquelme, et al., The Astrophys. Jour. 755, 50 (2012)