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Kinematic restrictions on particle collisions near extremal black holes

PHYSICS

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Introduction

In this work we explore the possibility of high-energy test particle collisions near horizons of extremal black holes, with emphasis on treating the previously known cases [1, 2] in an unifying, generalising manner. The main aim of the investigation is to study kinematic restrictions that may prevent the fine-tuned particles required for the high-energy collisions from reaching the near-horizon part of the spacetime. Before demonstrating these restrictions on the class of extremal Kerr-Newman black holes, we discuss their dependence on an arbitrary black hole model, starting from a general metric

$$\boldsymbol{g} = -N^2\,\mathbf{d}t^2 + g_{\varphi\varphi}\left(\mathbf{d}\varphi - \omega\,\mathbf{d}t\right)^2 + g_{rr}\,\mathbf{d}r^2 + g_{\vartheta\vartheta}\,\mathbf{d}\vartheta^2$$

accompanied by an electromagnetic potential

$$\boldsymbol{A} = A_t\,\mathbf{d}t + A_\varphi\,\mathbf{d}\varphi = -\phi\,\mathbf{d}t + A_\varphi\left(\mathbf{d}\varphi - \omega\,\mathbf{d}t\right) \ .$$

Horizons are given by zeroes of N^2 and in the extremal case, we use factorisation $N^2 = (r-r_0)^2\,\tilde{N}^2$. Equations of motion for charged test particles can be derived from the symmetries using canonical formalism just like in classical mechanics. Defining "forwardness" as

$$\mathscr{X} = \varepsilon - \omega l - \tilde{q}\phi \ ,$$

we can write

$$u^t = \frac{\mathscr{X}}{N^2} \ , \qquad u^\varphi = \frac{\omega\mathscr{X}}{N^2} + \frac{l - \tilde{q}A_\varphi}{g_{\varphi\varphi}} \ .$$

If we reduce the dimensionality by considering just the motion occurring in the equatorial hypersurface ($\vartheta = \pi/2$), third first order equation of motion follows from the velocity normalisation:

$$u^r = \pm\sqrt{\frac{1}{N^2 g_{rr}}\left[\mathscr{X}^2 - N^2\left(1 + \frac{\left(l - \tilde{q}A_\varphi\right)^2}{g_{\varphi\varphi}}\right)\right]} \ .$$

Parameters $\varepsilon, l, \tilde{q}$ represent the energy, axial angular momentum and charge of the particle (per unit rest mass). The most convenient way to study qualitative features of the motion is to use a close analogy of a classical potential defined by

$$V = \omega l + \tilde{q}\phi + N\sqrt{1 + \frac{\left(l - \tilde{q}A_\varphi\right)^2}{g_{\varphi\varphi}}} \ .$$

Then the condition for the motion to be allowed in the $r \geqslant r_+$ region becomes simply $\varepsilon \geqslant V$.

Particle collisions and critical particles

Centre-of-mass collision energy for two particles is an invariant given by

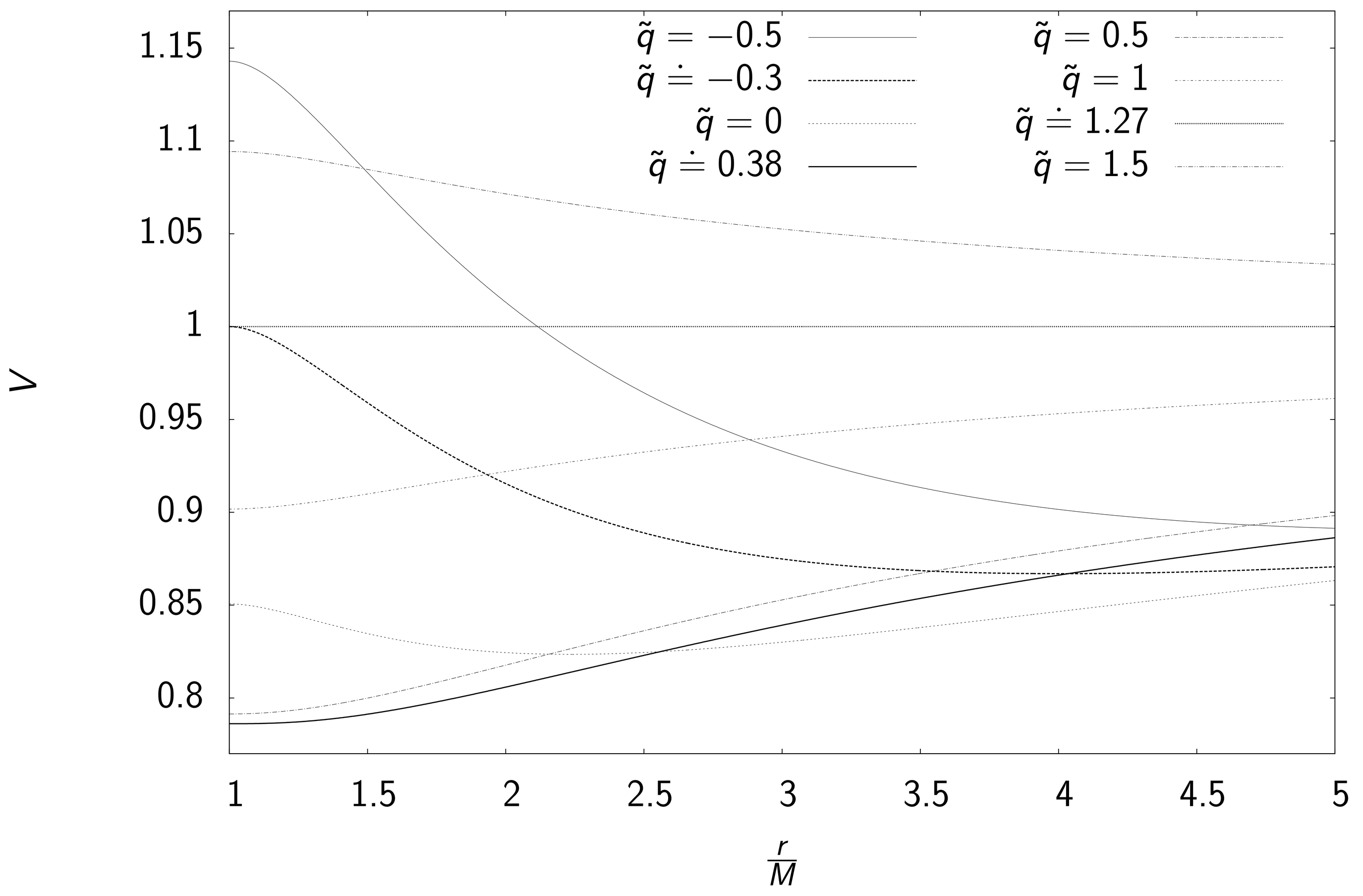
$$\frac{E_{\mathrm{CM}}^2}{2m_1m_2} = \frac{m_1}{2m_2} + \frac{m_2}{2m_1} - g_{\iota\kappa}u^\iota_{(1)}u^\kappa_{(2)} \ .$$

Using the expressions for test-particle velocity components in our black hole spacetime, and calculating the $N \to 0$ limit of the collision energy, which is

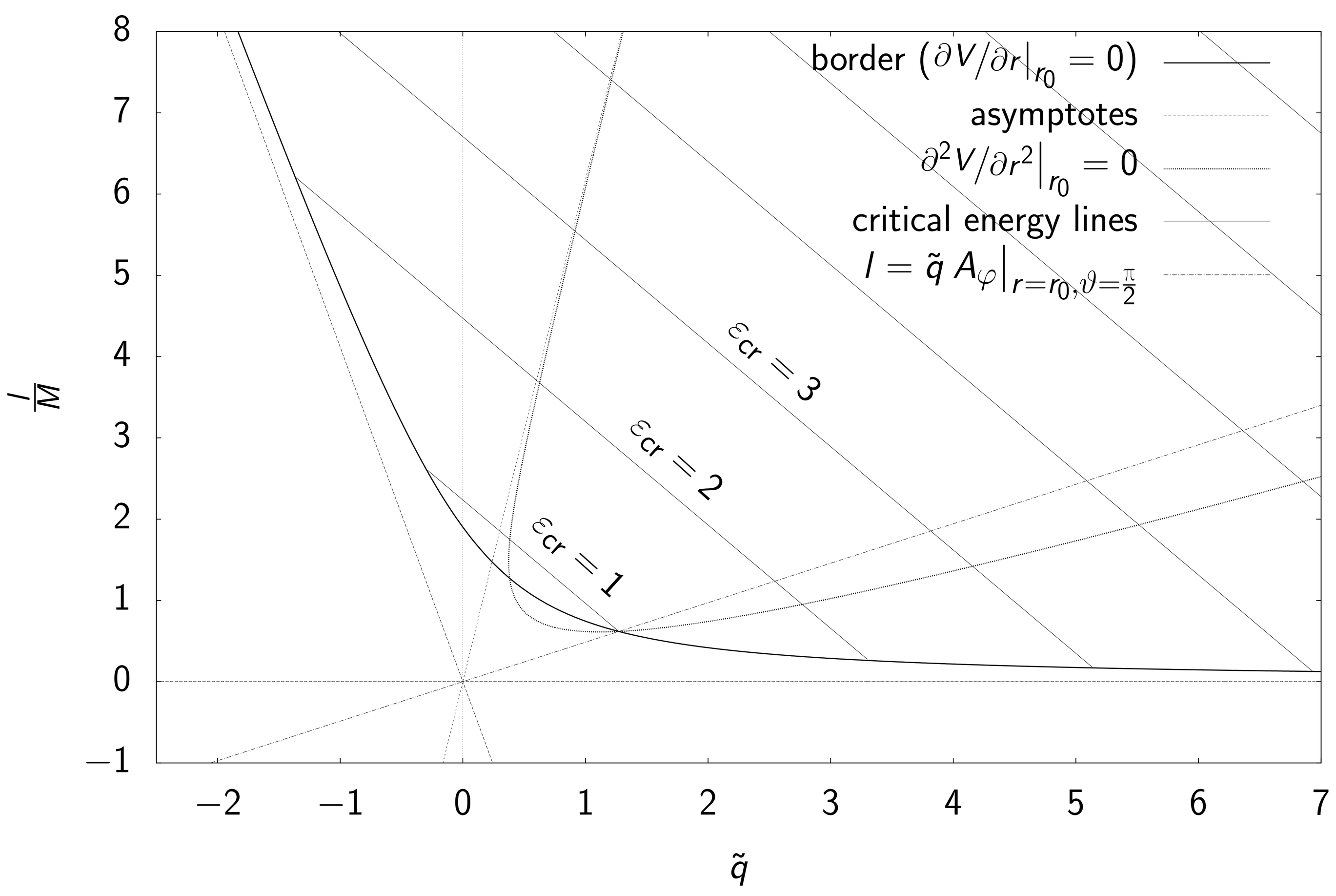
$$\left.\frac{E_{\mathrm{CM}}^2}{2m_1m_2}\right|_{N=0} = \frac{m_1}{2m_2} + \frac{m_2}{2m_1} - \left.\frac{\left(l_1 - \tilde{q}_1A_\varphi\right)\left(l_2 - \tilde{q}_2A_\varphi\right)}{g_{\varphi\varphi}}\right|_{N=0} + \\ + \frac{1}{2}\left.\left[1 + \frac{l_1 - \tilde{q}_2A_\varphi}{g_{\varphi\varphi}}\right]\right|_{N=0}\frac{\mathscr{X}_1^{\mathrm{H}}}{\mathscr{X}_2^{\mathrm{H}}} + \frac{1}{2}\left.\left[1 + \frac{l_1 - \tilde{q}_1A_\varphi}{g_{\varphi\varphi}}\right]\right|_{N=0}\frac{\mathscr{X}_2^{\mathrm{H}}}{\mathscr{X}_1^{\mathrm{H}}} \ ,$$

we can deduce that it will not be finite if one of the particles has a zero forwardness on the horizon ($\mathscr{X}^{\mathrm{H}} = 0$), whereas the other does have a non-zero one. Particles with $\mathscr{X}^{\mathrm{H}} = 0$ are called critical.

Curves of V for particles with $\left.\partial V/\partial r\right|_{r_0} = 0$ in the field of "Golden black hole"



Kinematic restrictions for critical particles in the field of "Golden black hole"



Kinematics of critical particles

We can observe that $\mathscr{X}^{\mathrm{H}} = 0$ implies $V \to \varepsilon$ for $r \to r_+$. Therefore, in order for the motion of critical particles towards r_+ to be allowed, V must decrease from its value $\varepsilon_{\mathrm{cr}}$ at r_+ for $r > r_+$. It turns out that for subextremal black holes the fist derivative of V at r_+ always goes to (plus) infinity, so the motion of critical particles towards r_+ is never allowed in this case. For degenerate horizons, denoted by r_0, the motion may or may not be allowed, depending on the parameters of the particle and also on the properties of the black hole model in question. These results generalise the same observations in [1, 2]. In order to understand in general such interplay between model dependence and dependence on parameters $\tilde{q}, l$ of the critical particle, we study the condition

$$\left.\frac{\partial V}{\partial r}\right|_{r=r_0} = 0$$

as a prescription for a curve in variables $\tilde{q}, l$. It turns out that it is just a branch of the following hyperbola:

$$\left.\left\{l^2\left[\frac{\left(\frac{\partial\omega}{\partial r}\right)^2}{\tilde{N}^2} - \frac{1}{g_{\varphi\varphi}}\right] + \tilde{q}^2\left[\frac{\left(\frac{\partial\phi}{\partial r}\right)^2}{\tilde{N}^2} - \frac{A_\varphi^2}{g_{\varphi\varphi}}\right] + 2l\tilde{q}\left(\frac{\frac{\partial\omega}{\partial r}\frac{\partial\phi}{\partial r}}{\tilde{N}^2} + \frac{A_\varphi}{g_{\varphi\varphi}}\right)\right\}\right|_{r=r_0,\vartheta=\frac{\pi}{2}} = 1 \ .$$

This curve makes division between the critical particles in the $\tilde{q}, l$ plane that can approach r_0 and those that can not. In the figure above, it is plotted for the example of "Golden black hole" (extremal Kerr-Newman black hole with M/a being the golden ratio, i.e. $\left(\sqrt{5}+1\right)/2$), where the hyperbola is aligned with $\tilde{q}$ axis. In the figure to the left, we plotted the effective potential V for various values of $\tilde{q}, l$ lying on this curve. It is clear that the second derivative of V at r_0 makes a big difference, when the first one is small or zero. Therefore, we turn to further condition

$$\left.\frac{\partial^2 V}{\partial r^2}\right|_{r=r_0} = 0 \ ,$$

which defines another, more complicated, curve in variables $\tilde{q}, l$. Its parametric expressions are

$$l = -\left.\left[\frac{\lambda\frac{\partial^2\phi}{\partial r^2}\sqrt{1 + \frac{\lambda^2A_\varphi^2}{g_{\varphi\varphi}}} + 2\frac{\partial\tilde{N}}{\partial r} + \left(2\frac{\partial\tilde{N}}{\partial r} - \frac{\tilde{N}}{g_{\varphi\varphi}}\frac{\partial g_{\varphi\varphi}}{\partial r} + 2\frac{\tilde{N}}{A_\varphi}\frac{\partial A_\varphi}{\partial r}\right)\frac{\lambda^2A_\varphi^2}{g_{\varphi\varphi}}}{\left(\frac{\partial^2\omega}{\partial r^2}A_\varphi + \frac{\partial^2\phi}{\partial r^2}\right)\sqrt{1 + \frac{\lambda^2A_\varphi^2}{g_{\varphi\varphi}}} + 2\tilde{N}\frac{\lambda A_\varphi}{g_{\varphi\varphi}}\frac{\partial A_\varphi}{\partial r}}A_\varphi\right]\right|_{r=r_0,\vartheta=\frac{\pi}{2}}$$

$$\tilde{q} = \left.\frac{\lambda\frac{\partial^2\omega}{\partial r^2}A_\varphi\sqrt{1 + \frac{\lambda^2A_\varphi^2}{g_{\varphi\varphi}}} - 2\frac{\partial\tilde{N}}{\partial r} - \left(2\frac{\partial\tilde{N}}{\partial r} - \frac{\tilde{N}}{g_{\varphi\varphi}}\frac{\partial g_{\varphi\varphi}}{\partial r}\right)\frac{\lambda^2A_\varphi^2}{g_{\varphi\varphi}}}{\left(\frac{\partial^2\omega}{\partial r^2}A_\varphi + \frac{\partial^2\phi}{\partial r^2}\right)\sqrt{1 + \frac{\lambda^2A_\varphi^2}{g_{\varphi\varphi}}} + 2\tilde{N}\frac{\lambda A_\varphi}{g_{\varphi\varphi}}\frac{\partial A_\varphi}{\partial r}}\right|_{r=r_0,\vartheta=\frac{\pi}{2}} \ .$$

This curve is also plotted in the above figure, and comparing with the figure on the left, we can confirm that only a short stretch of the $\left.\partial V/\partial r\right|_{r_0}$ curve lies in the region with $\left.\partial^2 V/\partial r^2\right|_{r_0} > 0$.

References

[1] Zaslavskii, Oleg B. *Acceleration of particles as a universal property of rotating black holes.* Physical Review D **82**, 083004 (2010).

[2] Zaslavskii, Oleg B. *Acceleration of particles by nonrotating charged black holes?* JETP Letters **92**, 571-574 (2011).