PhD **Open Days**

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Quasilinear approach to ray propagation in turbulent media

APPLAuSE – Advanced Program in Plasma Science and Engineering

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Introduction

Problem and motivation

In turbulent media, random fluctuations of the refractive index affect beam/ray propagation, causing, for instance, beam wandering and scattering, irradiance fluctuations, decrease of the spatial and temporal coherence, or beam spreading. These effects have a major impact in many applications, from free space communications to astronomy and propagation of RF waves in fusion plasmas.

Framework: geometrical optics/ray tracing

The use of ray tracing to describe wave propagation in turbulent atmospheres was pioneered by Tatarski and Chernov (1960's). It is a method that requires low computational effort and yields a straightforward physical interpretation. It can be used to easily assess physical problems, and can serve as a benchmark to more complex beam-tracing or full-wave simulations. Its validity becomes questionable when wave effects play a significant role along the wave propagation. Nonetheless, it has proven to be very useful to describe wave propagation in tokamak plasmas, where it has been widely used.

Objective and methodology

- To obtain the expressions for the average rays propagating in a turbulent media, and their dispersion caused by density fluctuations:
  - For that, we use a formal treatment, which provides a new approach, without recourse to Monte Carlo methods to perform ensemble averages. We apply an asymptotic analysis, keeping terms up to second order in the level of fluctuations.
- To apply the developed formalism to the simple case of optical rays in homogeneous turbulent media, using a single mode turbulence profile:
  - We have used Mathematica® to integrate a closed system of ordinary differential equations.
  - We compare the results with a Monte Carlo method.

Main equations

$$D(\omega,\mathbf{r},\mathbf{k}) = \omega - \omega(\mathbf{r},\mathbf{k}) = 0$$

$$\frac{dr_i}{dt} = \frac{\partial\omega}{\partial k_i} \qquad \frac{dk_i}{dt} = -\frac{\partial\omega}{\partial r_i}$$

Expansion in powers of the fluctuating terms up to 2nd order

$$\omega \simeq \omega_0 + \omega_1\delta n_e + \omega_2\delta n_e^2$$

Definition of the fluctuating terms

$$n_e = \overline{n}_e(\mathbf{r}) + \delta n_e(\mathbf{r})$$

$$\overline{\delta n_e}(\mathbf{r}) = 0$$

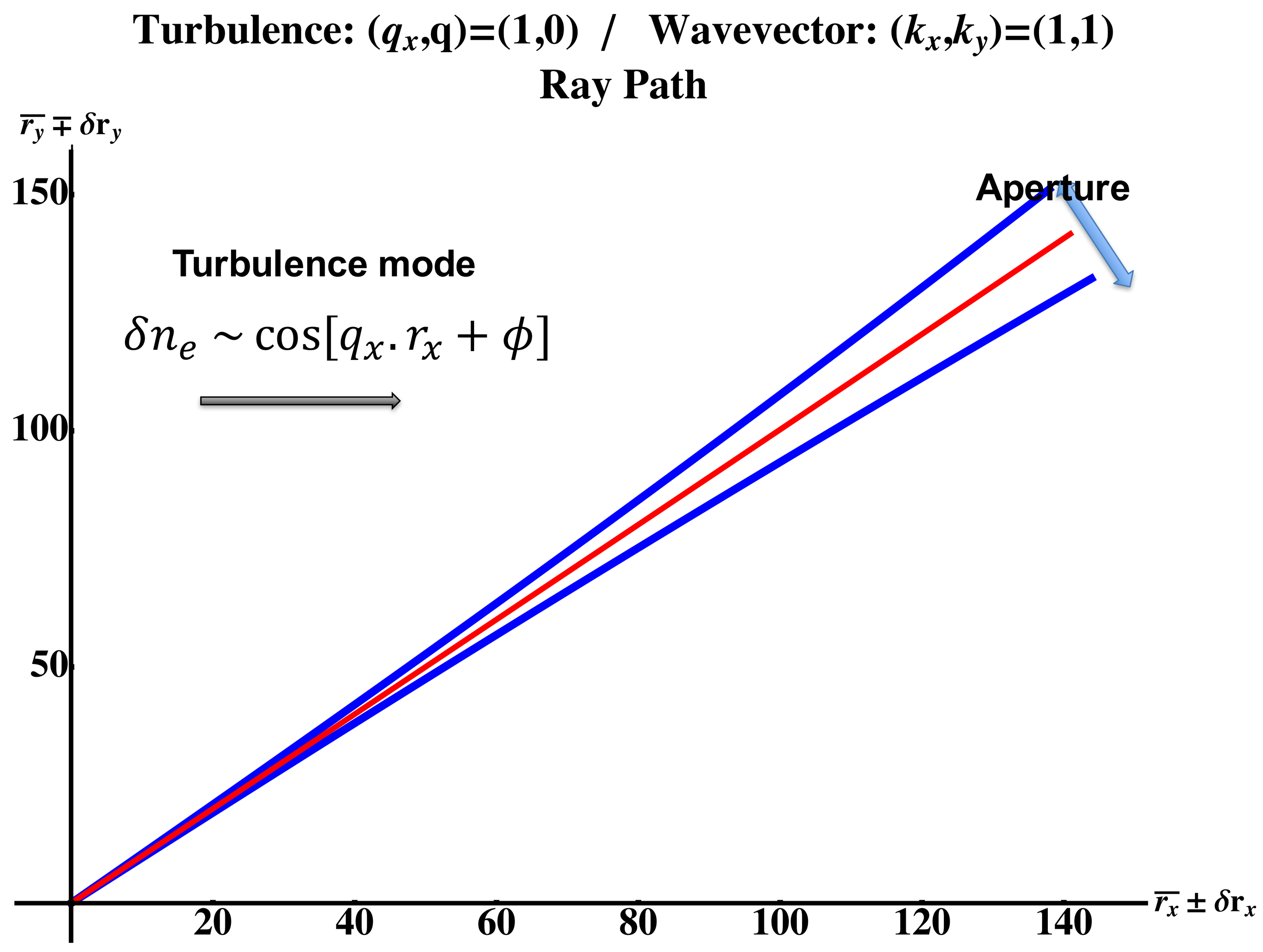
$$r_i = \overline{r}_i + \delta r_i \qquad k_i = \overline{k}_i + \delta k_i$$

Ensemble averaging

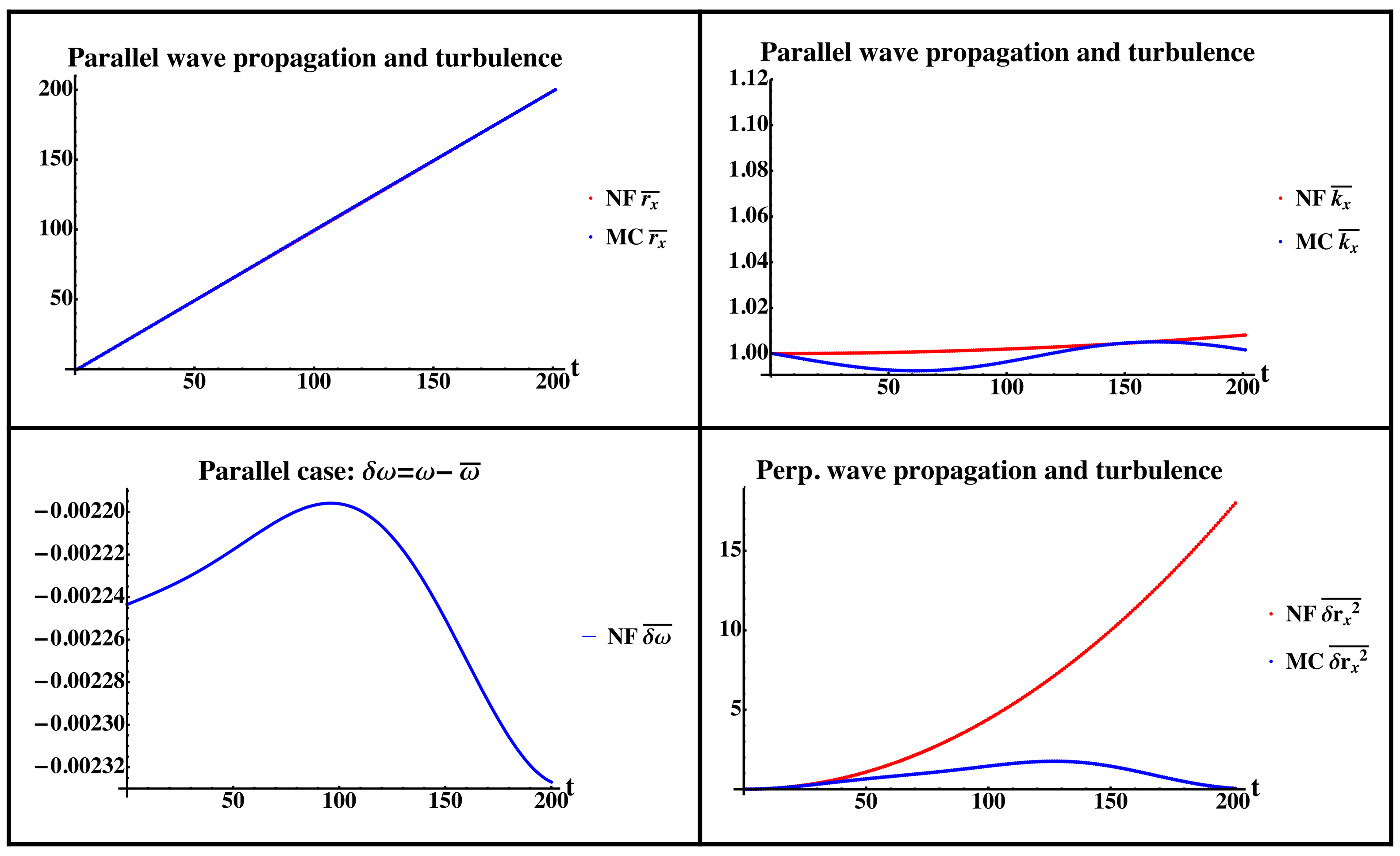
$$\frac{d\overline{r_i}}{dt} = \overline{\frac{\partial\omega}{\partial k_i}} \qquad \frac{d\delta r_i}{dt} = \frac{dr_i}{dt} - \frac{d\,\overline{r_i}}{dt}$$

$$\frac{d\overline{k_i}}{dt} = -\overline{\frac{\partial\omega}{\partial r_i}} \qquad \frac{d\delta k_i}{dt} = \frac{dk_i}{dt} - \frac{d\,\overline{k_i}}{dt}$$

Overview of the formalism: in the upper left, the dispersion relation is written in a form such that the ray tracing equations for the position **r** and wave vector **k** can be put in canonical form, using the frequency ω as Hamiltonian; in the lower left, we show how the variables are split into an average plus a fluctuating component. The dispersion relation is expanded in powers of the fluctuating term δn_e (top right). The expressions for the average rays and their dispersion are obtained by expanding the main equations in powers of the fluctuating terms δn_e, δr, and δk up to second order (bottom right).



Ray trajectory in a homogeneous media with a single mode turbulence profile in the x direction. The average ray trajectory is in red and its spreading due to turbulence is In blue. The latter is a result of having kept the 2nd order terms in the formal treatment.



Comparison between the present new formalism (NF) and Monte Carlo (MC): for the wave propagating parallel to the turbulence mode we show the average distance $\overline{r_x}$, average wave vector $\overline{k_x}$ and the Hamiltonian fluctuation $\delta\omega$. for the perpendicular case we show the average distance we show the spatial correlations $\overline{\delta r_x \delta r_x}$. Both methods show good agreement.

Conclusions

We have developed a new approach to integrate the ray equations in turbulent media, which is alternative to Monte Carlo and consists in a system of ODE's that is closed but infinite, so it may need to be truncated.

We this approach we have tested the formalism with a simple case of optical rays in homogeneous turbulent media, with a single turbulent mode profile. The results show good agreement with a Monte Carlo calculation. The effects of turbulence in the ray trajectories are visible and come from expanding the fluctuating quantities up to 2nd order, in a way that is similar to quasilinear theory.

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