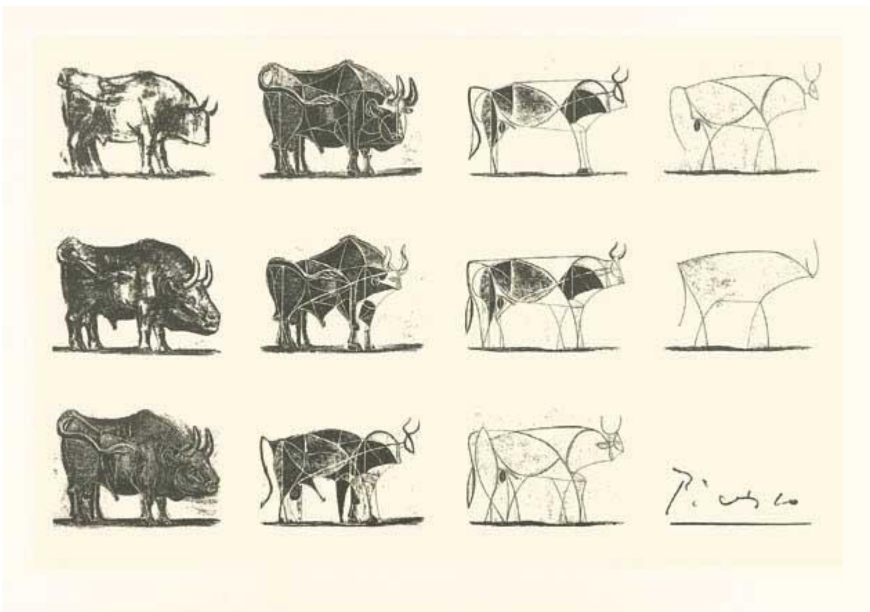
PhD Open Days



Quantum Markov networks

Recoverability of quantum states from direct correlations

Doctoral program in physics and mathematics of information (DPPMI)

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Overview

What is a quantum Markov network

Graphical models, such as Bayesian networks or Markov random fields, are widely used to represents classical systems of random variables in order of infer the most likely probability distribution (PD) that describes them.

Density operators (DOs) describes multipartite quantum systems and encompass PDs. A **quantum Markov network** is a possible graphical model for quantum systems that generalizes Markov random fields.

Why a quantum Markov network

Markov random field encode random variables and direct correlations between them. The graphical structure automatically evidences the conditional independent conditions that results in a factorization of the joint estimator that maximizes the entropy of the system thanks to Hammersley Clifford Theorem.

A quantum Markov networks would encode the bipartite correlations between the subparts of a quantum system and provide some learning techinque for compressing the computationally demanding information contained in a multipartite quantum state.

The restriction to bipartite correlations, other than being a widely used approximation in condense matter, also reduce exponentially the needed resources for the measurements.

Problem statement

Given a quantum system $\{X_1, \dots X_n\}$ and having access to its set of bipartite marginals $\{\rho_{X_iX_j} \in \mathcal{L}\left(\mathcal{H}_{X_iX_j}\right), i \neq j \in \{1, \dots, n\}\}$
to determine an efficient procedure for learning the maximum entropy estimator

$$\widetilde{\rho}_{\mathcal{X}} = \frac{1}{Z} \exp\left(\sum_{i,j} \sum_{\substack{k=0 \\ k^2+l^2 \neq 0}}^{d^2_{X_i}-1} \sum_{l=0}^{d^2_{X_j}-1} \lambda_{kl}^{(X_iX_j)} \Lambda_k^{(X_i)} \Lambda_l^{(X_j)} \right)$$

Where $\{\lambda_{jk}^{(X_iX_j)}\}$ are the Lagrange multipliers.

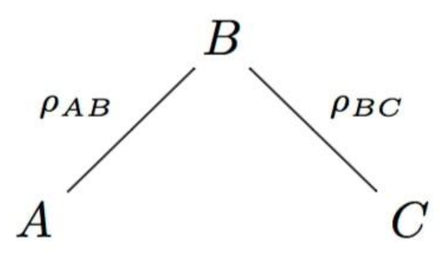
Computational complexity restriction: tree

Since DOs encompass PDs, we expect the complexity results of the classical analoguous problem to be recovered. Classically, the only graph structre known to result in an efficient recovery of the maximum entropy PD is the tree. (Dasgupta 1999)

We start then restricting our analysis to tree structured set of quantum bipartite marginals.

S.Di Giorgio, B. Mera, P.Mateus arxiv.org/pdf/1902.10087.pdf

Tripartite scenario



Definition: A quantum system ABC is called quantum Mark chain A-B-C when exists a CPTP map $\mathcal{R}_{B\to BC}$

$$\rho_{ABC} = (\mathcal{I}_A \otimes \mathcal{R}_{B\to BC})(\rho_{AB})$$

Theorem 1

A quantum Markov chain A-B-C maximizes the von Neumann entropy given two of its bipartite marginals and is efficiently and uniquely recoverable from them

$$\widetilde{\rho}_{ABC} = \rho_{BC}^{\frac{1}{2}} \rho_B^{-\frac{1}{2}} \rho_{AB} \rho_B^{-\frac{1}{2}} \rho_{BC}^{\frac{1}{2}}.$$

Theorem 2: algebraic nec and suff condition for compatibility

Given two bipartite quantum states $\{\rho_{AB}, \rho_{BC}\}$ they are compatible with a QMC A-B-C iff And the operator $\mathrm{Tr}_A(\rho_{AB}) = \mathrm{Tr}_C(\rho_{BC})$ is normal. $\Theta_{ABC} = \rho_{BC}^{\frac{1}{2}} \rho_B^{-\frac{1}{2}} \rho_{AB}^{\frac{1}{2}}$

Theorem 3: Best two out of three

Given a tripartite quantum system ABC and knowing its bipartite correl $\{\rho_{AB}, \rho_{BC}, \rho_{AC}\}$
If all the couple of marginals are compatible with a QMC, the *most likely one* is the obtained discarding the marginal with lowest $I_\rho(X:Y)$.

Multipartite scenario

Definition: quantum Markov tree

A quantum system on $\mathcal{H}_{\mathcal{X}} = \mathcal{H}_{X_1} \otimes \dots \otimes \mathcal{H}_{X_n}$ is said to be a a quantum Markov tree if for all i=1,…,n-2 :

$$I_\rho(X_{l_i} : V_i \backslash \{X_{l_i}, \mathrm{ad}(X_{l_i})\} | \mathrm{ad}(X_{l_i}))$$

Theorem 4

A quantum Markov tree on $\mathcal{H}_{\mathcal{X}} = \mathcal{H}_{X_1} \otimes \dots \otimes \mathcal{H}_{X_n}$ maximizes the von Neumann entropy constrained on the provided set of bipartite marginalsis and is algebraically recoverable from them. $\{\rho_{X_iX_j} \in \mathcal{L}\left(\mathcal{H}_{X_iX_j}\right), i \neq j \in \{1, \dots, n\}\}$

Algebraic nec and suff condition for compatibility

Given an quantum system $\{X_1, \dots X_n\}$nd knowing a tree structured set of its bipartite marginals, it is compatible with a quantum Markov tree iff

$$I_\rho\left(X_k : \mathrm{ad}\, Y_k | Y_k\right) = 0,\ Y_{k>1}, \mathrm{ad}\, Y_{k>1} \in \{X_n, X_1, \dots, X_{k-1}\}$$

Theorem 5: the best Markov tree

Given a quantum system $\{X_1, \dots X_n\}$ and knowing its complete set of bipartite marginals, if all the overlapping couples O(n^2) are compatible with a QMC, than the *most likely tree* is the one obtained from the subset:

$$\sum_{\{X_i, X_j\} \in E(\mathcal{T})} I_\rho(X_i, X_j)$$